TEST PAPER – 1

Mathematics – XI

Time : 3 hr   Max Marks : 100

GENERAL INSTRUCTIONS :-

1. *All questions are compulsory*.
2. *SECTION – A comprises of 6 questions of one marks each.*
3. *SECTION – B comprises of 13 questions of four marks each.*
4. *SECTION – C comprises of 7 questions of six marks each.*
5. *Internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.*

SECTION – A

Q. 1. Find the number of all possible relations that can be defined from the set {1, 2, 3} on the set {4, 5}.

Q. 2. If $y = \tan x^o$, find $\frac{dy}{dx}$

Q. 3. Find $\angle$ C if in triangle ABC the three sides are $a = 3$, $b = 5$, $c = 7$.

Q. 4. Find the vertex of the conic $y = 6x - x^2$.

Q. 5. If 'A' and 'B' are any two events such that P(A) = 0.42, P(B) = 0.48 & P(A∩B) = 0.16. Determine P(A but not B)

Q. 6. If the sum of n terms of an A.P. is $3n^2 + 5n$ then which term would be 164.

SECTION – B

Q. 7. A college awarded 38 medals in football, 15 in basketball and 20 in cricket. If these medals went to a total of 58 men and only three men got medals in all the three sports, how many received medals in exactly two of the three sports ?

Q. 8. By using the principle of mathematical induction prove that,

for all $n \geq 1$, $1 + \frac{1}{1+2} + \frac{1}{1+2+3} + - - - + \frac{1}{1+2+3+---+n} = \frac{2n}{n+1}$

Q. 9. Let R be a relation from N to N defined by R = $\{(a, b) : a, b \in \text{N and } a = b^2\}$. Are the following true?
(*i*) $(a, a) \in$ R, for all $a \in$ N   (*ii*) $(a, b) \in$ R, $\Rightarrow (b, a) \in$ R   (*iii*) $(a, b) \in$ R, $(b, c) \in$ R $\Rightarrow (a, c) \in$ R.

Q. 10. Solve the system of inequalities graphically: $x + 2y \leq 10$, $x + y \geq 1$, $x - y \leq 0$, $x \geq 0$, $y \geq 0$

Q. 11. In how many ways can the letters of the word *PERMUTATIONS* be arranged if the
(*i*) All vowels are not together   (*ii*) there are always 5 letters between P and S?

OR

If the different permutations of all the letter of the word *EXAMINATION* are listed as in a dictionary,
(*i*) How many words are there in this list before the first word starting with E?
(*ii*) How many words are there in this list starting with a vowel?

Q. 12. Prove that the middle term of the expansion $(1 + 4x + 4x^2)^n$ *is* $\frac{1.3.5.7.---.(2n-1)4^n.\ x^n}{n!}$; $n \in Z_+$

Q. 13. If p, q, r are in G.P. and the equations, $px^2 + 2qx + r = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then show that $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in A.P

OR

If p^{th}, q^{th}, r^{th} and s^{th} terms of an A.P are in G.P, then show that $(p - q)$, $(q - r)$, $(r - s)$ are also in G.P.

Q. 14. Find the equation of hyperbola having foci (± 4 , 0) and the length of latus rectum is 12.

Q. 15. Verify that (–1, 2, 1), (1, –2, 5), (4, –7, 8) and (2, –3, 4) are the vertices of a parallelogram and not of a rectangle.

P.T.O

Q. 16. Find the direction in which a line must be drawn through the point (–1 , 2) so that its point of intersection with the line $x + y = 4$ may be at a distance 3 units from this point .

OR

Find the image of the point (3, 8) with respect to the line $x + 3y = 7$ assuming the line to be a plane mirror.

Q. 17. Evaluate : $lim_{x \to 0} \left\{\dfrac{1 - cos\, x \sqrt{cos\, 2x}}{x^2}\right\}$

Q. 18. Prove that, $sin^3 x + sin^3\left\{x + \dfrac{2\pi}{3}\right\} + sin^3\left\{x + \dfrac{4\pi}{3}\right\} = -\dfrac{3}{4} sin\, 3x$

OR

If in a triangle ABC, $\dfrac{b+c}{12} = \dfrac{c+a}{13} = \dfrac{a+b}{15}$, prove that $\dfrac{cos\, A}{2} = \dfrac{cos\, B}{7} = \dfrac{cos\, C}{11}$

Q. 19. A typical *PIN* is a sequence of any four symbols chosen from the 26 letters in the alphabet and the ten digits. What is the probability that a randomly chosen *PIN* contains a repeated symbol?

SECTION – C

Q. 20. In a triangle ABC prove that : $(b - c)\, cot\left(\dfrac{A}{2}\right) + (c - a)\, cot\left(\dfrac{B}{2}\right) + (a - b)\, cot\left(\dfrac{C}{2}\right) = 0$

OR

Find the value of $sin\left(\dfrac{x}{2}\right)$, $cos\left(\dfrac{x}{2}\right)$ and $tan\left(\dfrac{x}{2}\right)$ if $sin\, x = \dfrac{1}{4}$; x lies in IIrd quadrant .

Q. 21. If $a + ib = \dfrac{3}{2 + cos\, x + i\, sin\, x}$, prove that $a^2 + b^2 = 4a - 3$

OR

Convert the complex number, $\dfrac{i - 1}{cos\left(\frac{\pi}{3}\right) + i\, sin\left(\frac{\pi}{3}\right)}$, into polar form

Q. 22. If 4-digit numbers greater than or equals to 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when,
(*i*) the digits are repeated ? (*ii*) the repetition of digits is not allowed ?

Q. 23. A line is such that its segment between the lines $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point (1, 5). Obtain its equation.

Q. 24. Find the equation of conic – section such that, e = $\dfrac{3}{4}$, foci on $y - axis$, centre at origin and passing through the point (6 , 4).

OR

Find the equation of the hyperbola having foci on $(0 , \pm\sqrt{10})$ and which passes through (2 , 3).

Q. 25. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in each of the following cases:
(*i*) If wrong item is omitted. (*ii*) If it is replaced by 12.

Q. 26. Find the differential coefficient of $sin\, \sqrt[3]{x}$, by first principle.

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