# TEST PAPER – 3
# Mathematics – XI

Time : 3 hr | Max Marks : 100

GENERAL INSTRUCTIONS :-

1. *All questions are compulsory.*
2. *SECTION – A comprises of 6 questions of one marks each.*
3. *SECTION – B comprises of 13 questions of four marks each.*
4. *SECTION – C comprises of 7 questions of six marks each.*
5. *Internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.*

## *SECTION – A*

Q. 1. Find P(P(P($\phi$))). If P(A) denotes the power set of the set A.

Q. 2. If $x \in [-2\ \ 1]$, then find the range of function $f(x) = \frac{1}{1+x^2}$.

Q. 3. If $y = e^{\sqrt{\tan x}}$, find $\frac{dy}{dx}$.

Q. 4. Find the probability that in a class three student all the three have different birthday.

Q. 5. Sum the series to infinity $x(x+y) + x^2(x^2+y^2) + x^3(x^3+y^3) + \text{- - -}$ with $|x| < 1$ and $|y| < 1$

Q. 6. Find the middle term(s) in the expansion $(9 - 6x^3 + x^6)^{12}$

## *SECTION – B*

Q. 7. Find the equation of the conic whose focus is (1, –1), directrix is the line $x - y = 3$ and $e = 0.5$

Q. 8. Using section formula, prove that the three points A(– 4, 6, 10), B(2, 4, 6) and C(14, 0, –2) are collinear. Also find the ratio in which C divides AB.

Q. 9. Find $\frac{dy}{dx}$, if (*i*) $y = \frac{x}{\sin^n x}$ (*ii*) $y = 2\tan x - 7\sec x$

OR

Using first principle, find the differential coefficient of $f(x) = \frac{2x+3}{x-2}$.

Q. 10. Prove that area of the triangle formed by the lines $y = m_1 x + c_1$ ; $y = m_2 x + c_2$ ; $x = 0$ is $\frac{(c_1 - c_2)^2}{2|m_1 - m_2|}$

Q. 11. If *a, b, c* are in A.P. and *b, c, d* are in G.P. and $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P. prove that *a, c, e* are in G.P.

OR

The ratio of the A.M. and G.M. of two positive numbers *a* and *b*, is *m* : *n*. Show that $a : b = m + \sqrt{m^2 - n^2} : m - \sqrt{m^2 - n^2}$

Q. 12. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that

(*i*) The student has opted neither NCC nor NSS. (*ii*) The student has opted NSS but not NCC.

Q. 13. How many words, with or without meaning, each of 3 vowels and 2 consonants can be formed from the letters of the word INVOLUTE ?

Q. 14. Solve the system of inequalities graphically: $2x + y \geq 4$, $x + y \leq 3$, $2x - 3y < 6$, $x \geq 0$, $y \geq 0$

Q. 15. Find the general solution of the equation ; $\tan 2x = -\cot\left\{x + \frac{\pi}{3}\right\}$.

OR

In a triangle ABC prove that : $a(\cos C - \cos B) = 2(b-c)\cos^2\left(\frac{A}{2}\right)$

Q. 16. If A $\subset$ B then prove the following (*i*) A – B = $\phi$ (ii) A $\cap$ B = A

Q. 17. In a triangle ABC prove that : $(b^2 - c^2)\cot A + (c^2 - a^2)\cot B + (a^2 - b^2)\cot C = 0$

P.T.O

Q. 18. Four Students *A, B, C, D* have given a question to solve. If *A* is twice as likely to solve the problem as *B*, and *B* and *C* are given about the same chance of solving the problem, while *C* is twice as likely to solve the problem as *D*, what is the probability that the question is solved.

Q. 19. If $(x + iy)^3 = u + iv$ , then show that $\frac{u}{x} + \frac{v}{y} = 4\,(x^2 - y^2)$ .

OR

Find the real numbers 'x' & 'y' if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$ .

## *SECTION – C*

Q. 20. The second, third and fourth terms in the binomial expansion $(x + a)^n$ are 240, 720 and 1080, respectively. Find '*x*', '*a*' and '*n*'.

Q. 21. A variable line which always remains at a constant distance '3*p*' from origin, cuts the coordinate axes at A, B respectively. Prove that the locus of the centroid of the triangle OAB is $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{p^2}$ .

OR

Find the equation of the straight line passing through the point (–2 , –7) and having an intercept of length three between the straight lines $4x + 3y = 12$ and $4x + 3y = 3$.

Q. 22. Find the value of '*a*' and '*b*', so that $\lim_{x \to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$, for the function

$$f(x) = \begin{cases} \dfrac{1 - \sin^3 x}{3\cos^2 x} & ; x < \dfrac{\pi}{2} \\ a & ; x = \dfrac{\pi}{2} \\ \dfrac{b\,(1 - \sin x)}{(\pi - 2x)^2} & ; x > \dfrac{\pi}{2} \end{cases}$$

Q. 23. Using principle of mathematical induction prove that for all natural $n \geq 1$ , $5^n - 5$ is a multiple of 4,

Hence, prove that for all natural $n \geq 1$, $2.7^n + 3.5^n - 5$ is divisible by 24.

Q. 24. Find the domain and range of the function $f(x) = \frac{2}{3}\sqrt{9 - x^2}$.

Also plot the rough sketch( graph) of the given function.

Q. 25. In a school, 30% of the student has 100% attendance. Previous year result report tells that 70% of all students having 100% attendance attain A grade and 10% of remaining students attain A grade in their annual examination. At the end of the year, One student is chosen at random find the probability that he has an A grade.

Q. 26. The mean and variance of eight observations are 9 and 9.25, respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

OR

The mean and standard deviation of a group of 100 observations were found to be 20 and 3, respectively. Later on it was found that three observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations are omitted.

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