

TEST PAPER – 1 (D. A. V. Special)

Mathematics – XI

Time : 3 hr

Max Marks : 100

General Instructions :

1. All questions are compulsory.
2. The question paper consists of **29 questions** divided into three sections **A, B** and **C**. **Section A** comprises of **10 questions of one mark** each, **Section B** comprises of **12 questions of four marks** each and **Section C** comprises of **07 questions of six marks** each.
3. All questions in **Section A** are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in **04 questions of four marks** each and **02 questions of six marks** each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

Section – A

- (1) Find the range of the function $f(x) = \sqrt{9 - x^2}$
- (2) Find the value of the complex number : $i^n + i^{n+1} + i^{n+2} + i^{n+3}$.
- (3) If length of perpendicular from origin on a line is **2 unit** and the perpendicular is making an angle **135°** with $x - axis$. Find equation of the line.
- (4) Evaluate : $\lim_{x \rightarrow 0} \left\{ \frac{e^{bx} - e^{ax}}{x} \right\}$
- (5) Find the derivative of : $x^3 e^x \sin x$
- (6) The number lock of a suitcase has **4 wheels**, each labelled with ten digits *i.e.*, from **0** to **9**. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?
- (7) If **A** and **B** are events such that $P(A) = 0.42$, $P(B) = 0.48$ and $P(A \text{ and } B) = 0.16$. Determine $P(A \text{ but not } B)$.
- (8) In the statement “*you are wet when it rains or you are in a river*”. Check whether “Or” used in the compound statement is exclusive or inclusive?
- (9) Write the negation of the statement “*there exists a rational number x such that, $x^2 = 2$* ”.
- (10) Is the statement ‘*Mumbai is the capital city of Maharashtra or Karnataka*’ true? Justify your answer.

P.T.O

Section – B

(11) Let **A** and **B** be sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set **X**, show that **A = B**.

(12) Show that, $\cos^2 x + \cos^2 \left\{ x + \frac{\pi}{3} \right\} + \cos^2 \left\{ x - \frac{\pi}{3} \right\} = \frac{3}{2}$.

OR

Find the value of $\tan \left(\frac{\pi}{8} \right)$

(13) Let **R** be a relation on **Z** defined by $R = \{(x, y) : |x - y| \text{ is divisible by } n ; x, y, n \in Z\}$.

Are the following true?

- (i) $(x, x) \in R$, for all $x \in Z$ (ii) $(x, y) \in R \Rightarrow (y, x) \in R$ (iii) $(x, y) \in R, (y, z) \in R \Rightarrow (x, z) \in R$.

(14) If '**α**' and '**β**' are two different complex numbers with $|\beta| = 1$, then find $\left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right|$

(15) In how many ways can the letters of the word **PERMUTATIONS** be arranged if the

- (i) vowels are all together, (ii) there are always 4 letters between **P** and **S**?

OR

From a class of 25 students, 10 are to be chosen for an excursion party. There are 3 students who decide that either all of them will join or none of them will join. In how many ways can the excursion party be chosen?

(16) In a triangle **ABC** prove that : $\sin \left(\frac{B - C}{2} \right) = \frac{b - c}{a} \cos \left(\frac{A}{2} \right)$

(17) If **p, q, r** are in **G.P.** and the equations, $px^2 + 2qx + r = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then show that $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in **A.P.**

(18) Prove that the product of the lengths of the perpendiculars drawn from the points $(\sqrt{a^2 - b^2}, 0)$ and $(-\sqrt{a^2 - b^2}, 0)$ to the line $\frac{x \cos \theta}{a} + \frac{y \sin \theta}{b} = 1$ is b^2

OR

Find the image of the point (3, 8) with respect to the line $x + 3y = 7$ assuming the line to be a plane mirror.

(19) Find the equation of the circle which passes through the point (4, 1) and (6, 5) and whose centre lies on the line $4x + y = 16$.

OR

Find the equation of the hyperbola with foci $(0, \pm \sqrt{10})$ and passing through the point (2, 3)

(20) Verify that (-1, 2, 1), (1, -2, 5), (4, -7, 8) and (2, -3, 4) are the vertices of a parallelogram and not of a rectangle.

(21) Using first principle, find the derivative of the function $f(x) = \sqrt{\tan x}$

OR

Evaluate : $\lim_{x \rightarrow \frac{\pi}{6}} \left\{ \frac{\sqrt{3} \sin x - \cos x}{x - \frac{\pi}{6}} \right\}$

(22) If '**A**', '**B**' and '**C**' are any three events associated with any random experiment, then prove that, $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$.

Section – C

(23) In a survey of **60** people, it was found that **25** people read newspaper **H**, **26** read newspaper **T**, **26** read newspaper **I**, **9** read both **H** and **I**, **11** read both **H** and **T**, **8** read both **T** and **I**, **3** read all three newspapers.

Find: (i) the number of people who read at least one of the newspapers.

(ii) the number of people who read exactly one newspaper.

(iii) the number of people who read exactly two newspaper.

(24) In a triangle **ABC** prove that : $(b^2 - c^2) \cot A + (c^2 - a^2) \cot B + (a^2 - b^2) \cot C = 0$

(25) Using principle of mathematical induction prove that,

$$\text{For all } n \geq 1, \frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \dots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

(26) If **a** and **b** are the roots of $x^2 - 3x + p = 0$ and **c**, **d** are roots of $x^2 - 12x + q = 0$, where **a**, **b**, **c**, **d** form a **G.P.** Prove that $(q + p) : (q - p) = 17:15$.

OR

$$\text{Show that : } \frac{1 \times 2^2 + 2 \times 3^2 + 3 \times 4^2 + \dots + n \times (n+1)^2}{1^2 \times 2 + 2^2 \times 3 + 3^2 \times 4 + \dots + n^2 \times (n+1)} = \frac{3n+5}{3n+1}$$

(27) Solve the following inequations graphically:

$$2x + y \leq 12, 4x + 5y > 20, x + 2y \leq 12, x \geq 0, y \geq 0$$

(28) The mean and standard deviation of **20** observations are found to be **10** and **2**, respectively. On rechecking, it was found that an observation **8** was incorrect. Calculate the correct mean and standard deviation in each of the following cases:

(i) If wrong item is omitted. (ii) If it is replaced by **12**.

(29) The coefficients of the $(r-1)^{\text{th}}$, r^{th} and $(r+1)^{\text{th}}$ terms in the expansion of $(x+1)^n$ are in the ratio **1 : 3 : 5**. Find '**n**' and '**r**'.

OR

(i) If the coefficients of a^{r-1} , a^r and a^{r+1} in the expansion of $(1+a)^n$ are in **A.P**, prove that $n^2 - n(4r+1) + 4r^2 - 2 = 0$.

(ii) Find **n**, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $(2^{1/4} + 3^{-1/4})^n$ is $\sqrt{6} : 1$.
