

TEST PAPER – 2 (D. P. S. Special)

Mathematics – XI

Time : 3 hr

Max Marks : 100

General Instructions :

1. All questions are compulsory.
2. The question paper consists of **29 questions** divided into three sections **A, B** and **C**. **Section A** comprises of **10 questions of one mark** each, **Section B** comprises of **12 questions of four marks** each and **Section C** comprises of **07 questions of six marks** each.
3. All questions in **Section A** are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in **04** questions of **four marks** each and **02** questions of **six marks** each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

- (1) Let $f = \{ (1, 1), (2, 3), (0, -1), (-1, -3) \}$ be a function from Z to Z defined by $f(x) = ax + b$, for some integers a, b . Determine a, b .
- (2) Express the complex number $(i^{18} + i^{-25})^3$, in the form of $a + ib$.
- (3) Find n ($P(P(P(\phi)))$)
- (4) Find 'n' if ${}^{n-1}P_3 : {}^nP_4 = 1 : 9$.
- (5) Evaluate : $\lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x)$
- (6) Find the length of perpendicular from $(2, -2, 4)$ on xy - plane.
- (7) A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is a vowel.
- (8) Evaluate : $\lim_{x \rightarrow 0} \left\{ \frac{x(e^{2+x} - e^2)}{1 - \cos x} \right\}$
- (9) Find the components of the statement 'p'
 p : A mixture of alcohol and water can be separated by chemical methods.
- (10) Find vertex of the parabola $x^2 + 2x + 2y + 3 = 0$.

P.T.O

Section – B

- (11) A survey shows that **63%** of Indians like coffee, whereas **76%** likes tea. If x % of Indians like both coffee and tea, find the range of possible values of x .
- (12) Let \mathbf{R} be a relation from \mathbf{N} to \mathbf{N} defined by $\mathbf{R} = \{(a, b) : a, b \in \mathbf{N} \text{ and } a = b^2\}$. Are the following true?
(i) $(a, a) \in \mathbf{R}$, for all $a \in \mathbf{N}$ (ii) $(a, b) \in \mathbf{R} \Rightarrow (b, a) \in \mathbf{R}$ (iii) $(a, b) \in \mathbf{R}, (b, c) \in \mathbf{R} \Rightarrow (a, c) \in \mathbf{R}$.
- (13) Show that, $\cos^2 x + \cos^2 \left\{x + \frac{\pi}{3}\right\} + \cos^2 \left\{x - \frac{\pi}{3}\right\} = \frac{3}{2}$.

OR

Find the value of $\tan \left(\frac{\pi}{8}\right)$

- (14) Sum the series to infinity $\frac{2}{5} + \frac{3}{5^2} + \frac{2}{5^3} + \frac{3}{5^4} + \dots$

- (15) Using principle of mathematical induction prove that, for all $n \geq 1$, $1^2 + 2^2 + 3^2 + \dots + n^2 > \frac{n^3}{3}$

- (16) If $y = \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}$, prove that, $2xy \frac{dy}{dx} = \frac{x}{a} - \frac{a}{x}$

OR

If $y = \sqrt{x} + \frac{1}{\sqrt{x}}$, prove that, $2x \frac{dy}{dx} + y = 2\sqrt{x}$

- (17) Out of **100** students, two sections of **40** and **60** are formed. If you and your friend are among **100** students. What is the probability that, (i) you both enter the same section?
(ii) you both enter the different section?

- (18) The coefficients of three consecutive terms in the expansion of $(1 + a)^n$ are in the ratio **1 : 7 : 42**. Find n .

OR

If the coefficients of a^{r-1} , a^r and a^{r+1} in the expansion of $(1 + a)^n$ are in **A.P**, prove that $n^2 - n(4r + 1) + 4r^2 - 2 = 0$.

- (19) Find the coordinates of the point where the line through $(3, -4, -5)$ and $(2, -3, 1)$ crosses the plane $2x + y + z = 7$
- (20) Find the equation of the circle which passes through the point $(2, -2)$ and $(3, 4)$ and whose centre lies on the line $x + y = 2$.
- (21) A ray of light passing through the point $(1, 2)$ reflects on the x – axis at the point **A** and the reflected ray passes through point $(5, 3)$, then find the coordinate of the point **A**.

OR

If origin is shifted to the point $(1, 1)$ find the transformed equation of $x^2 + y^2 - xy - x - y + 1 = 0$.

- (22) Find the mean deviation about the median for the following data:

x_i	3	6	9	12	13	15	21	22
f_i	3	4	5	2	4	5	4	3

Section – C

(23) Prove that : $a \cos A + b \cos B + c \cos C = 2a \sin B \sin C$

(24) Find the values of 'k' so that for the function $f(x) = \begin{cases} \frac{k \cdot \cos x}{\pi - 2x} & ; x \neq \frac{\pi}{2} \\ 3 & ; x = \frac{\pi}{2}, \end{cases}$

$$\lim_{x \rightarrow \pi/2} f(x) = f(\pi/2).$$

(25) Solve the inequation graphically: $2x + 3y \geq 6$, $x - 2y \leq 2$, $3x + 2y < 12$, $2y - 3x \leq 3$, $x \geq 0$, $y \geq 0$

(26) Prove that $\cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \frac{5\pi}{8} + \cos^2 \frac{7\pi}{8} = \frac{3}{2}$

(27) Let **S** be the sum, **P** the product and **R** the sum of reciprocals of n terms in a **G.P.** Prove that $P^2 R^n = S^n$.

OR

The ratio of the **A.M.** and **G.M.** of two positive numbers **a** and **b**, is $m : n$. Show that

$$a : b = m + \sqrt{m^2 - n^2} : m - \sqrt{m^2 - n^2}$$

(28) Find the direction in which a line must be drawn through the point $(-1, 2)$ so that its point of intersection with the line $x + y = 4$ may be at a distance **3 units** from this point .

OR

Two consecutive sides of a parallelogram are $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one diagonal is $11x + 7y = 9$, find the equation of the other diagonal.

(29) Find the mean and standard deviation using short-cut method.

x_i	60	61	62	63	64	65	66	67	68
f_i	2	1	12	29	25	12	10	4	5
