

TEST PAPER – 12 (G. H. P. S. Spl.)

Mathematics – XI

Time : 3 hr

Max Marks : 100

General Instructions :

1. All questions are compulsory.
2. The question paper consists of **29 questions** divided into three sections **A, B** and **C**. **Section A** comprises of **10 questions of one mark** each, **Section B** comprises of **12 questions of four marks** each and **Section C** comprises of **07 questions of six marks** each.
3. All questions in **Section A** are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in **04 questions of four marks** each and **02 questions of six marks** each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

- (1) If ϕ is an empty set. Find $n(\mathbf{P}(\mathbf{P}(\phi)))$.
- (2) Let $f = \{ (1, 1), (2, 3), (0, -1), (-1, -3) \}$ be a function from \mathbf{Z} to \mathbf{Z} defined by $f(x) = ax + b$, for some integers a, b . Determine a, b .
- (3) Draw the graph of the function $f(x) = \sqrt{4 - x^2}$
- (4) Find the value of: $\tan\left(\frac{\pi}{12}\right)$
- (5) If $\sin x = \frac{3}{5}$, $\cos y = \frac{12}{13}$, where x and y both lie in second quadrant, find the value of $\sin(x + y)$.
- (6) Three coins are tossed together. Describe three events which are mutually exclusive and exhaustive
- (7) Find the value of 'a' and 'b' if $\{ i^{18} + i^{-25} \}^3 = a + ib$
- (8) Find the multiplicative inverse of the complex number $\sqrt{5} + 3i$.
- (9) In a **G.P.**, the **3rd** term is **24** and the **6th** term is **192**. Find the **10th** term.
- (10) Find the slope of the line, which makes an angle of **30°** with the positive direction of y-axis measured anticlockwise.

P.T.O

SECTION – B

- (11) There are **200** individuals with a skin disorder, **120** had been exposed to the chemical **C₁**, **50** to chemical **C₂**, and **30** to both the chemicals **C₁** and **C₂**.
Find the number of individuals exposed to (i) Chemical **C₁** but not chemical **C₂**
(ii) No Chemical

OR

A market research group conducted a survey of **1000** consumers and reported that **720** consumers liked product **A** and **450** consumers liked product **B**. By mistake they have no data regarding the number of consumers liked both products. What should they assume it as a result they won't be fired

- (12) The ratio of the **A.M.** and **G.M.** of two positive numbers **a** and **b**, is **m : n**. Show that
 $a : b = m + \sqrt{m^2 - n^2} : m - \sqrt{m^2 - n^2}$

- (13) Show that : $\frac{\sec 8x - 1}{\sec 4x - 1} = \frac{\tan 8x}{\tan 2x}$

- (14) Using principle of mathematical induction prove that
For all $n \geq 1$, $1^2 + 2^2 + 3^2 + \dots + n^2 > n^3 / 3$

OR

Using principle of mathematical induction prove that, for all $n \geq 1$ $10^n + 3 \cdot 4^{n+2} + 5$ is divisible by **9**.

- (15) A manufacturer has **600** litres of a **12%** solution of acid. How many litres of a **30%** acid solution must be added to it so that acid content in the resulting mixture will be more than **15%** but less than **18%**?

OR

A plumber can be paid under two schemes given as; **Scheme – I : Rs 600** and **Rs 50** per hour, **Scheme – II : Rs 170** per hour. If the job takes **n** hours, for what values of **n** does the **scheme I** gives the plumber better wages ?

- (16) How many natural number not exceeding **4321** can be formed with the digits **1, 2, 3, and 4**, if the digits can repeat?

- (17) Find the polar form of the complex number $\frac{i - 1}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$

- (18) Find the vertex, focus, latus rectum, axis and directrix of the parabola : $4y^2 + 12x - 20y + 67 = 0$

- (19) Find the equation of conic – section such that, $e = 3 / 4$, foci on **y – axis**, centre at origin and passing through the point **(6, 4)**.

- (20) A point **R** with x-coordinate **4** lies on the line segment joining the points **P (2, –3, 4)** and **Q (8, 0, 10)**. Find the coordinates of the point **R**.

OR

Find the equation of the set of points **P**, the sum of whose distances from **A (4, 0, 0)** and **B(– 4, 0, 0)** is equal to **10**.

- (21) Solve for **x** : $\tan 2x = -\cot \left\{ x + \frac{\pi}{3} \right\}$

- (22) If **4**-digit numbers greater than or equals to **5,000** are randomly formed from the digits **0, 1, 3, 5, and 7**, what is the probability of forming a number divisible by **5** when, the digits are repeated ?

SECTION – C

(23) In ΔABC , Prove that : $\cos^2 A + \cos^2 B + \cos^2 C + 2 \cos A \cdot \cos B \cdot \cos C = 1$

OR

Solve for 'x' : $\tan x + \tan (x + \pi / 3) + \tan (x + 2\pi / 3) = 3$

(24) The coefficients of three consecutive terms in the expansion of $(1 + a)^n$ are in the ratio **1 : 7 : 42** . Find **n**.

(25) A line is such that its segment between the lines $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point **(1, 5)**. Obtain its equation.

OR

Find the distance of the line $4x - y = 0$ from the point **P (4, 1)** measured along the line making an angle of **135°** with the positive x-axis.

(26) Calculate mean, Variance and Standard Deviation for the following distribution.

Classes	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
Frequency	3	7	12	15	8	3	2

(27) If **a** and **b** are the roots of $x^2 - 3x + p = 0$ and **c**, **d** are roots of $x^2 - 12x + q = 0$, where **a, b, c, d** form a **G.P.** Prove that $(q + p) : (q - p) = 17:15$.

(28) Evaluate the following: (i) $\lim_{y \rightarrow 0} \left\{ \frac{(x+y) \sec(x+y) - x \sec x}{y} \right\}$

(ii) $\lim_{x \rightarrow 0} \left\{ \frac{1 - \cos x \sqrt{\cos 2x}}{x^2} \right\}$

(29) (i) If $y = \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}$, prove that, $2xy \frac{dy}{dx} = \frac{x}{a} - \frac{a}{x}$

(ii) If $y = \frac{x^3 \sin x}{\cos x}$, find $\frac{dy}{dx}$
