TEST PAPER – 3 (D. A. V. Special)

Mathematics – XI

Time : 3 hr **Max Marks : 100**

General Instructions :

1. All questions are compulsory.
2. The question paper consists of **29 questions** divided into three sections **A, B** and **C.** **Section A** comprises of **10** questions of **one mark** each, **Section B** comprises of **12** questions of **four marks** each and **Section C** comprises of **07** questions of **six marks** each.
3. All questions in **Section A** are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in **04** questions of **four marks** each and **02** questions of **six marks** each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

(1) Let **A ={1, 2, 3, 4}, B = {1, 5, 9, 11, 15, 16}** and ***f*** **= {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}** Are the following true**?** ***f*** is a function from **A** to **B.** Justify your answer in each case.

(2) Find the conjugate of $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

(3) Find equation of the line at an inclination of 30° with $y-axis$ at **(0 , 3).**

(4) Evaluate : $\lim_{x\to 0}\left\{\frac{e^x - e^{\sin x}}{x-\sin x}\right\}$

(5) If $y = \frac{x^3 . \sin x}{\cos x}$, find $\frac{dy}{dx}$

(6) In a relay race there are five teams ***A, B, C, D*** and ***E.*** What is the probability that ***A, B*** and ***C*** are first three to finish**.**

(7) If four cards are drawn from a well – shuffled deck of **52** cards**.** What is the probability of obtaining three diamonds and one spade**?**

(8) Write the statements **“***the banana trees will bloom if it stays warm for a month***”** in the form **“***if-then***”.**

(9) Write converse of the statement **‘*p*’**
p : *A positive integer is prime only if it has no divisors other than 1 and itself .*

(10) Rewrite statement **‘*p*’** in the form **“***if and only if* **”**
“*if a quadrilateral is equiangular, then it is a rectangle and if a quadrilateral is a rectangle, then it is equiangular***”.**

P.T.O

Section – B

(11) Find the equation of hyperbola having foci **$(\pm 4, 0)$** and the length of latus rectum is **12 .**

OR

Find the equation of the ellipse, such that major axis is ***x – axis*,** centre is at origin and the ellipse passes through **$(4, 3)$** and **$(6, 2)$.**

(12) Using section formula**,** prove that the three points **A(– 4, 6, 10), B(2, 4, 6)** and **C(14, 0, –2)** are collinear. Also find the ratio in which **C** divides **AB.**

(13) Using first principle, find the derivative of the function $f(x) = \sin\sqrt{x}$

OR

Evaluate : $\lim_{x \to 1} \left\{ \frac{x-2}{x^2-x} - \frac{1}{x^3-3x^2+2x} \right\}$

(14) Prove that **:** $\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ = \frac{1}{16}$.

OR

(*i*) If $x + y = \frac{\pi}{4}$, prove that $(\cot x - 1)(\cot y - 1) = 2$. (*ii*) Prove that $\tan 70^\circ = \tan 20^\circ + 2\tan 50^\circ$

(15) A survey shows that **63%** of Indians like coffee, whereas **76%** likes tea. If **x %** of Indians like both coffee and tea, find the range of possible values of **x.**

(16) Find the domain and range of the function $f(x) = \sqrt{4x - x^2}$

(17) How many words**,** with or without meaning**,** each of **3** vowels and **2** consonants can be formed from the letters of the word **INVOLUTE ?**

OR

How many natural number not exceeding **4321** can be formed with the digits **1, 2, 3,** and **4,** if the digits can repeat**?**

(18) If in a triangle **ABC,** $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$ **;** prove that $\angle C = 60^\circ$

(19) If the first and the n^{th} term of a **G.P.** are **'a'** and **'b',** respectively, and if **'P'** is the product of **n** terms**,** prove that **;** $P^2 = (ab)^n$.

(20) If $(x + iy)^3 = u + iv$, then show that $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$.

(21) Find the direction in which a line must be drawn through the point **(–1 , 2)** so that its point of intersection with the line $x + y = 4$ may be at a distance **3 units** from this point **.**

OR

Prove that area of the triangle formed by the lines $y = m_1 x + c_1$; $y = m_2 x + c_2$; $x = 0$ is $\frac{(c_1 - c_2)^2}{2|m_1 - m_2|}$

(22) A fair coin is tossed **4** times a person win **Re1,** for each head and lose **Rs1.50** for each tail that turns up**.** From the sample space & calculate how many different amount of money the person can have after **4** tosses also calculate the probability of having each of these amount**.**

Section – C

(23) In a class of **60** students**, 30** opted for **NCC, 32** opted for **NSS** and **24** opted for both **NCC** and **NSS.** If one of these students is selected at random**,** find the probability that

(*i*) The student has opted neither **NCC** nor **NSS.**
(*ii*) The student has opted neither **NCC** nor **NSS.**
(*iii*) The student has opted **NSS** but not **NCC.**

(24) Show that **:** $\left\{1 + \cos\left(\frac{\pi}{8}\right)\right\}\left\{1 + \cos\left(\frac{3\pi}{8}\right)\right\}\left\{1 + \cos\left(\frac{5\pi}{8}\right)\right\}\left\{1 + \cos\left(\frac{7\pi}{8}\right)\right\} = \frac{1}{8}$

(25) Find the sum of the series up to n terms **:** $1^3 + \frac{1^3 + 2^3}{1 + 3} + \frac{1^3 + 2^3 + 3^3}{1 + 3 + 5} + - - - .$

OR

The ratio of the **A.M.** and **G.M.** of two positive numbers a and b is $m : n$. Show that
$a : b = m + \sqrt{m^2 - n^2} : m - \sqrt{m^2 - n^2}$

(26) The second, third and fourth terms in the binomial expansion $(x + a)^n$ are **240, 720** and **1080,** respectively. Find **'*x*', '*a*'** and **'*n*'.**

OR

(*i*) Show that the middle term in the expansion of $(1 + x)^{2n}$ is $\frac{1.3.5.7.- - -.(2n-1)2^n.\ x^n}{n!}$; $n \in Z_+$.

(*ii*) If a and b are distinct integers, using binomial theorem prove that $a - b$ is a factor of $a^n - b^n$, whenever n is a positive integer.

(27) In a survey of **60** people**,** it was found that **25** people read newspaper **H, 26** read newspaper **T, 26** read newspaper **I, 9** read both **H** and **I, 11** read both **H** and **T, 8** read both **T** and **I, 3** read all three newspapers**.**

Find**: (i)** the number of people who read at least one of the newspapers**.**
(ii) the number of people who read exactly one newspaper**.**
(iii) the number of people who read exactly two newspaper**.**

(28) By using the principle of mathematical induction prove that**,** for all $n \geq 1$, $n^3 + (n + 1)^3 + (n + 2)^3$ is divisible by **9.**

(29) The mean and variance of eight observations are **9** and **9.25,** respectively**.** If six of the observations are **6, 7, 10, 12, 12** and **13,** find the remaining two observations**.**

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