TEST PAPER – 3 (D. P. S. Special)

Mathematics – XI

Time : 3 hr **Max Marks : 100**

General Instructions :

1. All questions are compulsory.

2. The question paper consists of **29 questions** divided into three sections **A, B** and **C. Section A** comprises of **10** questions of **one mark** each, **Section B** comprises of **12** questions of **four marks** each and **Section C** comprises of **07** questions of **six marks** each.

3. All questions in **Section A** are to be answered in one word, one sentence or as per the exact requirement of the question.

4. There is no overall choice. However, internal choice has been provided in **04** questions of **four marks** each and **02** questions of **six marks** each. You have to attempt only one of the alternatives in all such questions.

5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

(1) Determine the range of the function $f(x) = \dfrac{x^2}{1 + x^2}$

(2) Find sets **A, B** and **C** such that $A \cap B$, $B \cap C$ and $A \cap C$ are non-empty sets and $A \cap B \cap C = \phi$

(3) If $C(n, 3) = C(n, 7)$, find $C(n, 2)$.

(4) Find the equation of the circle with radius **5** whose centre lies on x-axis and passes through the point **(2, 3).**

(5) Find the probability that when a hand of **7** cards is drawn from a well shuffled deck of **52** cards, it contains all kings**.**

(6) Find the conjugate of $\dfrac{(3 - 2i)(2 + 3i)}{(1 + 2i)(2 - i)}$

(7) Find the length of perpendicular from **(3 , – 2 , 4)** on *y – axis.*

(8) Find the derivative of $x^3 e^x \sin x$

(9) Evaluate : $\lim_{x \to 0} \left\{ \dfrac{e^x - e^{\sin x}}{x - \sin x} \right\}$

(10) Write the following statements in the form 'if-then'.
"You get a job implies that your credentials are good".

P.T.O

Section – B

(11) Find the equation of conic – section such that, $e = \frac{3}{4}$, foci on y – ***axis***, centre at origin and passing through the point **(6 , 4).**

(12) Using section formula, prove that the three points **A(– 4, 6, 10), B(2, 4, 6)** and **C(14, 0, –2)** are collinear. Also find the ratio in which **C** divides **AB.**

(13) Evaluate : $\lim_{x \to 1} \left\{ \frac{x-2}{x^2 - x} - \frac{1}{x^3 - 3x^2 + 2x} \right\}$ **OR** Evaluate : $\lim_{x \to 2} \left\{ \frac{3^x + 3^{3-x} - 12}{3^{3-x} - 3^{x/2}} \right\}$

(14) Prove that the product of the lengths of the perpendiculars drawn from the points $(\sqrt{a^2 - b^2}\ ,\ 0)$ and $(-\sqrt{a^2 - b^2}\ ,\ 0)$ to the line $\frac{x \cos\theta}{a} + \frac{y \cos\theta}{b} = 1$ is b^2

(15) Show that the coefficient of the middle term in the expansion of $(1 + x)^{2n}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1 + x)^{2n-1}$.

OR

If in the expansion of $(1 + x)^n$, the coefficient of 5^{th}, 6^{th} and 7^{th} terms are in **A.P.** Find n.

(16) In a relay race there are five teams ***A, B, C, D*** and **E.**
(a) What is the probability that ***A, B*** and ***C*** finish first, second and third, respectively.
(b) What is the probability that ***A, B*** and ***C*** are first three to finish (in any order)

OR

Two balls are drawn at random without replacement from a bag containing **2** white, **3** red, **5** green, **4** black balls. Find the probability that both the balls are of different colours.

(17) There are **200** individuals with a skin disorder, **120** had been exposed to the chemical C_1, **50** to chemical C_2, and **30** to both the chemicals C_1 and C_2. Find the number of individuals exposed to
(i) Chemical C_1 but not chemical C_2 ***(ii)*** No Chemical

(18) Find the domain and range of the function $f(x) = \sqrt{4x - x^2}$

(19) Prove that : $\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ = \frac{1}{16}$.

OR

(i) If $x + y = \frac{\pi}{4}$, prove that $(\cot x - 1)(\cot y - 1) = 2$. ***(ii)*** Prove that $\tan 70^\circ = \tan 20^\circ + 2 \tan 50^\circ$

(20) Using principle of mathematical induction prove that,

For all $n \geq 1$, $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \text{- - - } + n \text{ terms} = \frac{n(n+1)^2(n+2)}{12}$

(21) If ***p, q, r*** are in **G.P.** and the equations, $px^2 + 2qx + r = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then show that $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in **A.P**

(22) Find mean deviation about the mean for the following data :

x_i	2	5	6	8	10	12
f_i	2	8	10	7	8	5

Section – C

(23) In a triangle **ABC** prove that : $\frac{b^2 - c^2}{a^2} \sin 2A + \frac{c^2 - a^2}{b^2} \sin 2B + \frac{a^2 - b^2}{c^2} \sin 2C = 0$

(24) Show that : $\left\{1 + \cos\left(\frac{\pi}{8}\right)\right\}\left\{1 + \cos\left(\frac{3\pi}{8}\right)\right\}\left\{1 + \cos\left(\frac{5\pi}{8}\right)\right\}\left\{1 + \cos\left(\frac{7\pi}{8}\right)\right\} = \frac{1}{8}$

OR

In **Δ ABC,** Prove that **: sin(B + C – A) + sin(C + A – B) – sin(A + B – C) = 4.cosA.cosB.sinC**

(25) Find the distance of the point **(1 , 2)** from the line $4x + 7y + 5 = 0$ along the line $2x - y = 3$.

(26) ***(i)*** Solve the inequation : $\frac{2x - 1}{x + 1} < 1$; $x \in R$

(ii) A manufacturer has **600** litres of a **12%** solution of acid. How many litres of a **30%** acid solution must be added to it so that acid content in the resulting mixture will be more than **15%** but less than **18%?**

(27) Find the sum of the series up to ***n*** terms : $1^3 + \frac{1^3 + 2^3}{1 + 3} + \frac{1^3 + 2^3 + 3^3}{1 + 3 + 5} + - - - .$

(28) Using limit process find the derivative of the function $f(x) = x^2 \sin x$.

OR

Find the value of ***'a'***, so that $\lim_{x \to 0} f(x) = f(0)$, for the function

$$f(x) = \begin{cases} a \sin \frac{\pi(x+1)}{2} & ; x \leq 0, \\ \frac{\tan x - \sin x}{x^3} & ; x > 0 \end{cases}$$

(29) The mean and standard deviation of **20** observations are found to be **10** and **2,** respectively**.** On rechecking**,** it was found that an observation **8** was incorrect**.** Calculate the correct mean and standard deviation in each of the following cases **: (i)** If wrong item is omitted**.**
(ii) If it is replaced by **12.**

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