

TEST PAPER – 3 (St. Xavier)

Mathematics – XI

Time : 3 hr

Max Marks : 90

GENERAL INSTRUCTIONS :-

1. All questions are compulsory.
2. SECTION – A comprises of 6 questions of one marks each.
3. SECTION – B comprises of 12 questions of four marks each.
4. SECTION – C comprises of 6 questions of six marks each.
5. Internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

SECTION – A

- Q. 1. Find  $P(P(P(\phi)))$ . If  $P(A)$  denotes the power set of the set  $A$ .
- Q. 2. If  $x \in [-2, 1]$ , then find the range of function  $f(x) = \frac{1}{1+x^2}$ .
- Q. 3. If  $y = e^{\sqrt{\tan x}}$ , find  $\frac{dy}{dx}$ .
- Q. 4. Find the probability that in a class three student all the three have different birthday.
- Q. 5. Find the coordinates of a point on  $y$ -axis which are at a distance of  $5\sqrt{2}$  from the point  $(3, -2, 5)$ .
- Q. 6. Find the middle term(s) in the expansion  $(9 - 6x^3 + x^6)^{12}$

SECTION – B

- Q. 7. Find the equation of the ellipse whose focus is  $(1, -1)$ , directrix is the line  $x - y = 3$  and  $e = 0.5$
- Q. 8. Using section formula, prove that the three points  $A(-4, 6, 10)$ ,  $B(2, 4, 6)$  and  $C(14, 0, -2)$  are collinear. Also find the ratio in which  $C$  divides  $AB$ .
- Q. 9. Find  $\frac{dy}{dx}$ , if (i)  $y = \frac{x}{\sin^n x}$  (ii)  $y = 2\tan x - 7\sec x$

OR

Using first principle, find the differential coefficient of  $f(x) = \frac{2x+3}{x-2}$ .

- Q. 10. Prove that area of the triangle formed by the lines  $y = m_1 x + c_1$ ;  $y = m_2 x + c_2$ ;  $x = 0$  is  $\frac{(c_1 - c_2)^2}{2|m_1 - m_2|}$
- Q. 11. Four Students  $A, B, C, D$  have given a question to solve. If  $A$  is twice as likely to solve the problem as  $B$ , and  $B$  and  $C$  are given about the same chance of solving the problem, while  $C$  is twice as likely to solve the problem as  $D$ , what is the probability that the question is solved.
- Q. 12. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that  
(i) The student has opted neither NCC nor NSS. (ii) The student has opted NSS but not NCC.

OR

On her vacations Veena visits four cities  $(A, B, C$  and  $D)$  in a random order. What is the probability that she visits (i)  $A$  before  $B$ ? (ii)  $A$  just before  $B$ ?

- Q. 13. Solve the system of inequalities graphically:  $2x + y \geq 4$ ,  $x + y \leq 3$ ,  $2x - 3y < 6$ ,  $x \geq 0$ ,  $y \geq 0$

OR

How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

- Q. 14. In a triangle  $ABC$  prove that :  $a(\cos C - \cos B) = 2(b - c) \cos^2\left(\frac{A}{2}\right)$
- Q. 15. A survey shows that 63% of Indians like coffee, whereas 76% likes tea. If  $x$  % of Indians like both coffee and tea, find the range of possible values of  $x$ .
- Q. 16. If  $a$  and  $b$  are distinct integers, using binomial theorem prove that  $(a - b)$  is a factor of  $a^n - b^n$ , whenever  $n$  is a positive integer.

Q. 17. Find the equation of the circle passing through origin and making intercepts 'a' and 'b' on the coordinate axes.

OR

Find *focus*, *axis* the equation of the *directrix*, and length of the latus rectum of the parabola  $x^2 = -9y$ .

Q. 18. Evaluate :  $\lim_{x \rightarrow 1} \left\{ \frac{x-2}{x^2-x} - \frac{1}{x^3-3x^2+2x} \right\}$

SECTION – C

Q. 19. The coefficients of the  $(r-1)^{th}$ ,  $r^{th}$  and  $(r+1)^{th}$  terms in the expansion of  $(x+1)^n$  are in the ratio 1 : 3 : 5. Find 'n' and 'r'.

Q. 20. A variable line which always remains at a constant distance '3p' from origin, cuts the coordinate axes at A, B respectively. Prove that the locus of the centroid of the triangle OAB is  $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{p^2}$ .

OR

Find the equation of the straight line passing through the point  $(-2, -7)$  and having an intercept of length three between the straight lines  $4x + 3y = 12$  and  $4x + 3y = 3$ .

Q. 21. Find the value of 'a' and 'b', so that  $\lim_{x \rightarrow \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$ , for the function

$$f(x) = \begin{cases} \frac{1 - \sin^3 x}{3 \cos^2 x} & ; x < \frac{\pi}{2} \\ a & ; x = \frac{\pi}{2} \\ \frac{b(1 - \sin x)}{(\pi - 2x)^2} & ; x > \frac{\pi}{2} \end{cases}$$

Q. 22. Find the domain and range of the function  $f(x) = \frac{2}{3} \sqrt{9 - x^2}$ .

Also plot the rough sketch( graph) of the given function.

Q. 23. In a triangle ABC prove that :  $(b^2 - c^2) \cot A + (c^2 - a^2) \cot B + (a^2 - b^2) \cot C = 0$

Q. 24. The mean and variance of eight observations are 9 and 9.25, respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

OR

The mean and standard deviation of a group of 100 observations were found to be 20 and 3, respectively. Later on it was found that three observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations are omitted.

\*\*\*\*\*