TEST PAPER – 1
Mathematics – XII

Time : 3 hr   Max Marks : 100

GENERAL INSTRUCTIONS :-

1. *All questions are compulsory.*
2. *SECTION – A comprises of 4 questions of one marks each.*
3. *SECTION – B comprises of 8 questions of two marks each.*
4. *SECTION – C comprises of 11 questions of four marks each.*
5. *SECTION – D comprises of 6 questions of six marks each.*
6. *Internal choice has been provided in 03 questions of four marks each and 03 questions of six marks each. You have to attempt only one of the alternatives in all such questions.*

SECTION – A

Q. 1. Find the number of all possible binary operations defined over the set $\{a, b\}$.

Q. 2. Let $A = \{1, 2, 3\}$. Then find the number of relations containing $(1, 2)$ and $(1, 3)$ which are reflexive and symmetric but not transitive.

Q. 3. Find the number of all possible invertible matrices of order 2×2 with each entry 0 or 1.

Q. 4. Find the order of the differential equation for the family of the curve $y = ae^{x} + be^{x+c}; a, b, c \in Real$

SECTION – B

Q. 5. Find the value of $x + y + z$ if, $cos^{-1}x + cos^{-1}y + cos^{-1}z = 0$

Q. 6. Evaluate : $\int tan^{3}x \, dx$

Q. 7. If $y = tan^{-1}x$, prove that $\dfrac{dy}{dx} = \dfrac{1}{1 + x^2}$

Q. 8. Find both the maximum and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ in the interval $[0, 3]$.

Q. 9. If, A and B are symmetric matrices of same order, show that, $AB - BA$ is a skew-symmetric matrix.

Q. 10. Using E – Transformations find inverse of the matrix $\begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$

Q. 11. Find the distance between the line : $\dfrac{x-2}{3} = \dfrac{y-3}{4} = \dfrac{z-4}{2}$ and the plane $2x - y - z = 5$

Q. 12. Find area of the triangle having vertices $(1, 2, 3)$, $(0, 3, 5)$ and $(2, 3, 0)$.

SECTION – C

Q. 13. If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a\,(x - y)$ prove that $\dfrac{dy}{dx} = \dfrac{\sqrt{1 - y^2}}{\sqrt{1 - x^2}}$

OR

If $y = \left(x + \sqrt{1 + x^2}\right)^{m}$, then prove that $(1 + x^2)y_2 + x\,y_1 - m^2\,y = 0$

Q. 14. Prove that : $sin^{-1}\left(\dfrac{12}{13}\right) + cos^{-1}\left(\dfrac{4}{5}\right) + tan^{-1}\left(\dfrac{63}{16}\right) = \pi$.

Q. 15. Find the value of k so that the function $f(x) = \begin{cases} \dfrac{1 - cos\,4x}{x^2} & ; x < 0 \\ k & ; x = 0 \\ \dfrac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4} & ; x > 0 \end{cases}$

so that the function is continuous at $x = 0$

Q. 16. Evaluate : $\int (1 - x^2)dx \Big/ x(1 - 2x)$ OR   Evaluate : $\int_{-1}^{2} |x^3 - x| \, dx$

Q. 17. Define a binary operation $*$ on the set $\{0,1,2,3,4,5\}$ as: $a * b = \begin{cases} a+b & ; a+b<6 \\ a+b-6 & ; a+b \geq 6 \end{cases}$

Find the *identity element* and *inverse element* of this operation.

Q. 18. Find the equation of the normal(s) to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y = 4$

Q. 19. Solve the differential equation, $(tan^{-1}y - x)\, dy = (1 + y^2)dx$

Q. 20. Let $\vec{a} = 4i + 5j - k$, $\vec{b} = i - 4j + 5k$ and $\vec{c} = 3i + j - k$. Then find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and satisfying $\vec{c}.\ \vec{d} = 21$.

Q. 21. Find the vector equation of the line passing through $(1,2,3)$ and parallel to the planes $\vec{r}.\ (i - j + 2k) = 5$ and $\vec{r}.\ (3i + j + k) = 6$ **.**

Q. 22. There are three coins**.** One is a two headed coin **(** having head on both face **),** another is a biased coin that comes up heads 75 % of the time and third is an unbiased coin. One of the three coin is chosen at random and tossed, it shows heads, what is the probability that it was a two headed coin ?

Q. 23. In a hurdle race, a player has to cross ten hurdles. The probability that he clear each hurdle is $\frac{5}{6}$ .What is the probability that he will knock down fewer than two hurdles?

OR

A coin is biased so that the head is three times as likely to occur as tail. If the coin is tossed twice, find the expected number of tails per toss.

SECTION - D

Q. 24. Evaluate : $\int_0^{\pi/2} log\ (sin\, x)\ dx$

Q. 25. Using Properties of determinants prove that $\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (c+a)^2 & bc \\ ca & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

Q. 26. Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

OR

Find the volume of the largest cylinder that can be inscribed in a sphere of radius R.

Q. 27. Find the area lying above $x - axis$ & included between the circle $x^2 + y^2 = 8x$ and interior to the parabola $y^2 = 4x$.

OR

Find the area of region $\{(x,y): 0 \leq y \leq x^2 + 1, 0 \leq\ y \leq x + 1, 0 \leq x \leq 2\}$

Q. 28. Find the Cartesian as well as the vector equation of the plane through the intersection of the planes $\vec{r}.(i + 3j) + 6 = 0$ and $\vec{r}.(3i - j + 4k) = 0$**,** and at a unit distance from origin.

OR

If α , β , γ and δ are the angles made by any line with the four diagonals of any cube, then prove that, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$.

Q. 29. A young man rides his motorcycle at $25\ km/hr$, he has to spend ` $2/km$ on petrol, if he rides it at a faster speed of $40\ km/hr$, the petrol cost increases to ` $5/km$. He has ` 100 to spend on petrol and wishes to find the maximum distance he can travel within 1 hr. Express this as an L.P.P. and solve it.

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