TEST PAPER – 1

Mathematics – XII

Time : 3 hr | Max Marks : 100

GENERAL INSTRUCTIONS :-

1. *All questions are compulsory.*
2. *SECTION – A comprises of 4 questions of one marks each.*
3. *SECTION – B comprises of 8 questions of two marks each.*
4. *SECTION – C comprises of 11 questions of four marks each.*
5. *SECTION – D comprises of 6 questions of six marks each.*
6. *Internal choice has been provided in 03 questions of four marks each and 03 questions of six marks each. You have to attempt only one of the alternatives in all ssuch questions.*

SECTION – A

Q. 1. Find the number of all possible binary operations defined over the set $\{a, b\}$ with a as identity element.

Q. 2. Give example of two non – zero square matrices of order 2, whose product is a zero matrix.

Q. 3. Find the number of relations which are reflexive and transitive but not symmetric, defined over the set $\{1, 2, 3\}$ containing $(1, 2)$ and $(2, 3)$

Q. 4. Write down the all possible vectors of magnitude two in $YZ - plane$.

SECTION – B

Q. 5. Find the value of $x + y + z$ if, $sin^{-1}x + sin^{-1}y + sin^{-1}z = \frac{3\pi}{2}$

Q. 6. If $x - y = \frac{\pi}{2}$, find $\begin{bmatrix} cos^2 x & cos\,x.sin\,x \\ cos\,x.sin\,x & sin^2 x \end{bmatrix}\begin{bmatrix} cos^2 y & cos\,y.sin\,y \\ cos\,y.sin\,y & sin^2 y \end{bmatrix}$

Q. 7. Test the applicability of Converse of Rolle's Theorem over, $f(x) = x^3 - 5x^2 - 3x, x \in [0\ \ 4]$

Q. 8. If $(x + 1)e^y = 1$, show that $\left(\frac{dy}{dx}\right)^2 = \frac{d^2y}{dx^2}$

Q. 9. Find the anti derivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$

Q. 10. If $|\vec{a}| = a$ then find the value of $|\vec{a} \times i|^2 + |\vec{a} \times j|^2 + |\vec{a} \times k|^2$

Q. 11. Show that, $x^2 = 2y^2 \log y$ is the solution of the differential equation $(x^2 + y^2)\frac{dy}{dx} = xy$

Q. 12. Prove that the two events A & B are independent if, $P(\bar{A} \cap \bar{B}) = P(\bar{A})P(\bar{B})$.

SECTION – C

Q. 13. If $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ and $f(x) = x^2 + 4x + 7$, then find $f(A)$. Also find the matrix X if, $f(A) + X = O$

Q. 14. Find the value of a if the function $f(x) = \begin{cases} a \sin\left\{(x+1)\frac{\pi}{2}\right\} & ; x \le 0 \\ \frac{\tan x - \sin x}{x^3} & ; x > 0 \end{cases}$ is continuous at $x = 0$

Q. 15. If $y = \frac{sin^{-1}x}{\sqrt{1 - x^2}}$, then prove that $(1 - x^2)y_2 - 3xy_1 - y = 0$

OR

Differentiate $tan^{-1}\left\{\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right\}$ with respect to $cos^{-1}(x^2)$

Q. 16. Solve the differential equation, $(sin^{-1}y - x)\,dy = \sqrt{1 - y^2}\,dx$.

Q. 17. Find the interval in which the function $f(x) = \log(\cos x)$ is increasing and decreasing.

Q. 18. Prove that the perimeter of a right angled triangle of given hypotenuse is maximum, when the triangle is isosceles.

P.T.O

Q. 19. Evaluate : $\int (1 - x^2)dx \Big/ x(1 - 2x)$ <u>OR</u> Evaluate : $\int_{-1}^{2} |x^3 - x| \, dx$

Q. 20. Let $\vec{a} = 4i + 5j - k$, $\vec{b} = i - 4j + 5k$ and $\vec{c} = 3i + j - k$. Then find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and satisfying $\vec{c} . \vec{d} = 21$.

Q. 21. Find the equation of the line passing through the point (2, 4, –1) and perpendicular to the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-4}{-9}$.

Q.22. A coin is biased so that the head is three times as likely to occur as tail. If the coin is tossed twice, find the expected number of tails per toss.

<u>OR</u>

From the 21 tickets marked through 1 to 21. Three tickets are drawn at random. Find the probability that the number on the tickets drawn are in A.P.

Q. 23. Out of a group of 8 highly qualified doctors in a hospital, 6 are very kind and cooperative with their patients and so are very popular, while the other two remain reserved. For a health camp, three doctors are selected at random. Find the probability distribution of the number of very popular doctors. What values are expected from the doctors?

SECTION - D

Q. 24. A variable plane which is at the distance of '3*p*' from origin, cuts the coordinate axes at A, B and C. Then prove that the locus of the centroid of the triangle ABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$.

<u>OR</u>

Find the equation of the plane passing through intersection of the planes, $\vec{r} . (i - j) + 6 = 0$ and $\vec{r} . (3i + 3j - 4k) = 0$, and at a unit distance from origin.

Q. 25. Show that the ΔABC is an isosceles triangle if the determinant

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 + \cos A & 1 + \cos B & 1 + \cos C \\ \cos A + cos^2 A & \cos B + cos^2 B & \cos C + cos^2 C \end{vmatrix} = 0$$

Q. 26. Find the area lying above $x - axis$ & included between the circle $x^2 + y^2 = 8x$ and parabola $y^2 = 4x$.

<u>OR</u>

Find the area of region $\{(x, \ y) : 0 \le y \le x^2 + 3 \, , 0 \le y \le 2x + 3 \, , 0 \le x \le 3\}$

Q. 27. Evaluate : $\int_0^1 tan^{-1}(1 - x + x^2) \, dx$

Q. 28. Let A = N × N and ∗ be the binary operation on A defined by $(a , b) \ast (c , d) = (a + c , b + d)$.

Show that '∗' is commutative & associative. Find the identity element for '∗' on A if any.

<u>OR</u>

Let N br the set of all natural numbers and R be the relation on N × N defined by :
$(a, b)R(c, d) \Leftrightarrow ad(b + c) = bc(a + d)$. Show that relation R is an equivalence relation.

Q. 29. A dealer in rural area wishes to purchase a number of sewing machines. He has only Rs. 5760 to invest and has space for at most 20 items. A electronic sewing machine costs him Rs.360 and a manually operated sewing machine Rs. 240. He can sell an Electronic Sewing Machine at a profit of Rs. 22 and a manually operated sewing machine at a profit of Rs.18. Assuming that he can sell all the items that he can buy how should he invest his money in order to maximize his profit. Make it as a linear programming problem and solve it graphically. Keeping the rural background in mind justify the 'values' to be promoted for the selection of the manually operated machine.

******