TEST PAPER – 2
Mathematics – XII

Time : 3 hr Max Marks : 100

GENERAL INSTRUCTIONS :-

1. *All questions are compulsory.*
2. *SECTION – A comprises of 4 questions of one marks each.*
3. *SECTION – B comprises of 8 questions of two marks each.*
4. *SECTION – C comprises of 11 questions of four marks each.*
5. *SECTION – D comprises of 6 questions of six marks each.*
6. *Internal choice has been provided in 03 questions of four marks each and 03 questions of six marks each. You have to attempt only one of the alternatives in all ssuch questions.*

SECTION – A

Q.1. If *'f '* and *'g'* are two surjective functions then *'gof '* is surjective, but the converse need not be true. Justify your answer with a suitable example.

Q. 2. Write down all possible relation over the set $\{1, 2, 3\}$ containing (1, 2) and (2,3) which are only transitive.

Q. 3. If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix}$, Apply the transformation $C_2 \rightarrow C_2 + C_1$

Q. 4. Write the vector equation of the line : $2x - 3y = 0$, $z = 2$.

SECTION – B

Q. 5. Evaluate : $\int_{3\pi/8}^{5\pi/8} sin^6x\, cos^5x\, dx$

Q. 6. Prove that for any two vectors $\vec{a}$ and $\vec{b}$, $\left|\vec{a} \times \vec{b}\right|^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} \end{vmatrix}$

Q. 7. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

Q. 8. Evaluate : $cos^{-1}x - cos^{-1}\left\{\frac{x}{2} + \frac{1}{2}\sqrt{3 - 3x^2}\right\}$; $x \geq \frac{1}{2}$

Q. 9. If $x^m . y^n = (x + y)^{m+n}$ prove that $\frac{dy}{dx} = \frac{y}{x}$

Q. 10. Form the differential equation representing the family of curves $y = a \sin(x + b)$, where a, b are arbitrary constants.

Q. 11. Find the angle between the line : $\frac{2x-2}{6} = \frac{3-y}{-4} = \frac{3z-4}{6}$ and the plane $2x - 2y + z - 5 = 0$.

Q. 12. A car starts from a point P at time $t = 0$ seconds and stops at point Q. The distance x, in metres, covered by it, in t seconds is given by $x = t^2\left(2 - \frac{t}{3}\right)$. Find the distance between P and Q.

SECTION – C

Q.13. If $A = \begin{bmatrix} x & 2 & 1 \\ 2 & y & 3 \\ 1 & 3 & z \end{bmatrix}$, $xyz = 6, 9x + y + 4z = 23$, find $A(adj\ A)$.

Q.14. Show that the function $f(x) = |x - a|$ is continuous but not differentiable at $x = a$, $\forall\ x, a \in R$.

Q.15. If $y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{(x-c)} + 1$,

prove that, $\frac{dy}{dx} = \frac{y}{x}\left\{\frac{a}{(a-x)} + \frac{b}{(b-x)} + \frac{c}{(c-x)}\right\}$

Q.16. Show that the curves $xy = a^2$ and $x^2 + y^2 = 2a^2$ touch each other. Also find the point of contact.

P.T.O

Q.17. Find the intervals in which the function 'f' given by $f(x) = \dfrac{4\sin x - 2x - x\cos x}{2 + \cos x}$ is increasing & decreasing.

OR

Show that $y = \log(1 + x) - \dfrac{2x}{2 + x}$; $x > -1$; is an increasing function of x throughout its domain.

Q.18. Evaluate : $\int \sqrt{1 + x^2}\, dx$ OR $\int (2e^{3x} + 3e^{-4x})\, dx \Big/ (3e^{3x} + 4e^{-4x})$

Q.19. Find the point of intersection of the lines $\dfrac{x-1}{3} = \dfrac{y-1}{-1}$, $z = -1$; $\dfrac{x-4}{2} = \dfrac{z+1}{3}$, $y = 0$ if the lines are intersecting.

Q. 20. Form the differential equation representing the family of curves : $y = c_1 e^{ax} \cos bx + c_2 e^{ax} \sin bx$, with a and b are known constants while c_1 and c_2 are arbitrary constants.

Q. 21. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three mutually perpendicular vectors of equal magnitude, then find the angle made by the vector $\vec{a} + \vec{b} + \vec{c}$ with the above three vectors, hence show that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$

OR

If the vectors $\vec{a} = xi + j + k$, $\vec{b} = i + yj + k$ and $\vec{c} = i + j + z\,k$ are coplanar, then prove that $\dfrac{1}{1-x} + \dfrac{1}{1-y} + \dfrac{1}{1-z} = 1$, where $x, y, z \neq 1$

Q. 22. A person plays a game of tossing a coin thrice. For each head, he is given Rs 2 by the organiser of the game and for each tail, he has to give Rs 1.50 to the organiser. Fnd the expected gain / loss of the person per throw.

Q. 23. A man is to known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability the man is speaking truth. *Write any three benefits of speaking the truth*.

SECTION - D

Q. 24. Show that the relation R, in the set Z as $R = \{(a, b) : a \equiv b\ (3)\ ;\ a, b \in Z\}$, with $a \equiv b\ (3) \Rightarrow |a - b|$ is divisible by 3, is an equivalence relation. Also define three partitions of Z which are mutually disjoint.

Q.25. Find the distance of the point $(-2, 3, -4)$ from the line $\dfrac{x+3}{3} = \dfrac{2y+3}{4} = \dfrac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$

OR

Find the equation of the plane passing through the line of intersection of planes, $3x - 4y + 5z = 10$; $2x + 2y - 3z = 4$ and parallel to the line $x = 2y = 3z$.

Q.26. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit ?

Q.27. If $x + y + z = 0$, using properties of determinants prove that $\begin{vmatrix} ax & by & cz \\ cy & az & bx \\ bz & cx & ay \end{vmatrix} = xyz \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$.

Q. 28. Evaluate : $\int_0^{2\pi} dx \Big/ (e^{\cos x} + 1)$ OR Evaluate : $\int_{-1}^{1/2} |x \cos \pi x| dx$

Q. 29. Find the area of the region $\{(x, y) : x^2 + y^2 \leq 16\ ;\ y^2 \geq 6x\}$

OR

Find the area of the region bounded between the curves $y = \sqrt{5 - x^2}$, $y = |x - 1|$ and $x - axis$.

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