

TEST PAPER – 2
Mathematics – XII

Time : 3 hr

Max Marks : 100

GENERAL INSTRUCTIONS :-

1. All questions are compulsory.
2. SECTION – A comprises of 4 questions of one marks each.
3. SECTION – B comprises of 8 questions of two marks each.
4. SECTION – C comprises of 11 questions of four marks each.
5. SECTION – D comprises of 6 questions of six marks each.
6. Internal choice has been provided in 03 questions of four marks each and 03 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

SECTION – A

- Q. 1. Find the smallest and largest equivalence relation that can be defined over the set $\{1, 2, 3\}$
- Q. 2. Find the value of $i. (j \times k) + j. (k \times i) + k. (j \times i)$
- Q. 3. An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be at least 4 successes.
- Q. 4. If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix}$, Apply the transformation $C_2 \rightarrow C_2 + C_1$

SECTION – B

- Q. 5. Find the value of, $\cos^2(\tan^{-1}2) + \sin^2(\cot^{-1}2)$
- Q. 6. If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if $AB = BA$.
- Q. 7. Find the value of 'x' if $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = \mathbf{0}$
- Q. 8. Evaluate : $\int \left\{ 1/\log x - 1/(\log x)^2 \right\} dx$
- Q. 9. Using properties of determinants and without expanding prove that $\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$
- Q. 10. If $|\vec{a}| = a$ then find the value of $|\vec{a} \times i|^2 + |\vec{a} \times j|^2 + |\vec{a} \times k|^2$
- Q. 11. Find the angle between the line : $\frac{2x-2}{6} = \frac{3-y}{-4} = \frac{3z-4}{6}$ and the plane $2x-2y+z=5$.
- Q. 12. Using derivative find the approximate value of $\sqrt{0.6}$

SECTION – C

- Q. 13. Find the equation of normal to the curve $x^2 = 4y$ which passes through the point (1, 2)

OR

Find the intervals in which the function 'f' given by $f(x) = \frac{4\sin x - 2x - x \cos x}{2 + \cos x}$ is increasing & decreasing.

- Q.14. Find the point of intersection of the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$; $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$, if the lines are intersecting.

- Q. 15. Prove that : $\cos \left[\tan^{-1} \left\{ \sin \left(\cot^{-1} x \right) \right\} \right] = \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$

- Q. 16. Evaluate : $\int \sqrt{\tan x} dx$ OR Evaluate : $\int x^2 \sin^{-1} x . dx$

- Q.17. If \vec{a} , \vec{b} , \vec{c} are three mutually perpendicular vectors of equal magnitude, then find the angle made by the vector $\vec{a} + \vec{b} + \vec{c}$ with the above three vectors, hence show that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to \vec{a} , \vec{b} and \vec{c}
- Q. 18. Form the differential equation representing the family of curves : $y = c_1 e^{ax} \cos bx + c_2 e^{ax} \sin bx$, with a and b are known constants while c_1 and c_2 are arbitrary constants.

OR

Solve the differential equation, $xdy + ydx - xdx + xy \cot x dx = 0$; $x = 0, y = 1$

- Q. 19. Let R be the relation on $N \times N$ defined by, $(a, b)R(c, d) \Leftrightarrow ad = bc$.

Show that relation R is an equivalence relation.

- Q. 20. Show that the function $f(x) = |x - a|$ is continuous but not differentiable at $x = a, \forall x, a \in R$.

- Q. 21. Find $\frac{dy}{dx}$ if, $y = \sin^{-1} x + \sin^{-1} \sqrt{1 - x^2}$; $-1 \leq x \leq 1$

- Q. 22. Out of a group of 8 highly qualified doctors in a hospital, 6 are very kind and cooperative with their patients and so are very popular, while the other two remain reserved. For a health camp, three doctors are selected at random. Find the probability distribution of the number of very popular doctors.

- Q. 23. Assume that the chance of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?

SECTION - D

- Q. 24. An apache helicopter of enemy is flying along the curve $y = x^2 + 7$. A soldier placed at $(3, 7)$ wants to hit down the helicopter when it is at the minimum distance. Find the minimum distance.

OR

Find the volume of the largest cylinder that can be inscribed in a cone of height h and semi-vertical angle α

- Q. 25. Find the equation of the plane passing through the line of intersection of planes, $3x - 4y + 5z = 10$; $2x + 2y - 3z = 4$ and parallel to the line $x = 2y = 3z$.

- Q. 26. Evaluate : $\int_0^{\pi} \log(1 + \cos x) dx$

- Q. 27. Find the area of the region $\{ (x, y) : x^2 + y^2 \leq 16 ; y^2 \geq 6x \}$

OR

Find the area of the region enclosed between the two circles $x^2 + y^2 = 1$ and $(x - 1)^2 + y^2 = 1$.

- Q. 28. If $x \neq y \neq z$, and $D = \begin{vmatrix} x & x^2 & 1 + x^3 \\ y & y^2 & 1 + y^3 \\ z & z^2 & 1 + z^3 \end{vmatrix} = 0$, then show that $1 + xyz = 0$.

- Q. 29. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?
