TEST PAPER – 3

Mathematics – XII

Time : 3 hr   Max Marks : 100

GENERAL INSTRUCTIONS :-

1. *All questions are compulsory*.
2. *SECTION – A comprises of 4 questions of one marks each*.
3. *SECTION – B comprises of 8 questions of two marks each*.
4. *SECTION – C comprises of 11 questions of four marks each.*
5. *SECTION – D comprises of 6 questions of six marks each.*
6. *Internal choice has been provided in 03 questions of four marks each and 03 questions of six marks each. You have to attempt only one of the alternatives in all such questions.*

SECTION – A

Q. 1. Write the all possible equivalence relation(s) R on set $A = \{1, 2, 3\}$, containing $(1, 2)$.

Q. 2. If A is a square matrix of order 3 and $|adj(A)| = 144$, then find the value of $|3A|$

Q. 3. Find the angle between the two unit vectors $\vec{a}$ and $\vec{b}$ such that $\vec{a} + \vec{b}$ is also a unit vector.

Q. 4. Find the order and degree of the differential equation $y_2{}^3 + y_3{}^2 + \sin y_1 = \cos x$

SECTION – B

Q. 5. Find the range of $\cos^{-1} x$, $|x| \le \frac{1}{2}$

Q. 6. If foot of perpendicular from $(4, -1, 2)$ on a plane is $(-10, 5, 4)$. Then find equation of such plane.

Q. 7. Evaluate : $\int dx/\sqrt{e^{2x} - 1}$

Q. 8. If, $y = \cot^{-1}\sqrt{\cos x} - \tan^{-1}\sqrt{\cos x}$, prove that $\sin y = \tan^2\left(\frac{x}{2}\right)$.

Q. 9. If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$; prove that ; $\frac{dy}{dx} = \frac{1}{x^3 y}$.

Q. 10. If the vertices A, B, C of a triangle ABC are $(1, 2, 3), (-1, 0, 0), (0, 1, 2)$ respectively, then find $\angle B$.

Q. 11. Find the value of, $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3$

Q. 12. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

SECTION – C

Q. 13. Verify Rolle's Theorem for the function $f(x) = (x - 1)(x - 2)^2$, $x \in [1 \quad 2]$.

Q. 14. Using properties of determinants, prove that $\begin{vmatrix} a & b-c & b+c \\ c+a & b & c-a \\ a-b & a+b & c \end{vmatrix} = (a + b + c)(a^2 + b^2 + c^2)$.

Q. 15. If, $x\sqrt{1 + y} + y\sqrt{1 + x} = 0$; $-1 < x < 1$, then prove that $\frac{dy}{dx} = -\frac{1}{(1 + x)^2}$

Q. 16. If, $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$ and $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$ then prove that $\frac{dy}{dx} = -\cot 3t$

Q. 17. If R_1 and R_2 are two equivalence relations, then show that $R_1 \cap R_2$ is also an equivalence relation.

Q. 18. Evaluate: $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\, dx$ OR   Evaluate: $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}\, dx$

Q. 19. Solve the initial value problem : $\frac{dy}{dx} + y\cot x = 2x + x^2 \cot x$; $y\left(\frac{\pi}{2}\right) = 0$

OR

Show that the general solution of the differential equation $(x^2 + x + 1)dy + (y^2 + y + 1)\,dx = 0$ is given by $(x + y + 1) = a(1 - x - y - 2xy)$

Q. 20. Decompose the vector $5i - 2j + 5k$ into vectors which are parallel & perpendicular to $3i + k$.

Q. 21. Find the distance of the point (–1, –5, –10) from the point of intersection of the line $\vec{r} = 2i - j + 2k + \lambda\,(3i + 4j + 2k)$ and the plane $\vec{r}.\,(i - j + k) = 5$.

OR

Find the image of the point (1, 3, 4) in the plane $2x - y + z + 3 = 0$.

Q. 22. In a group of 100 families, 30 families like male child, 25 families like female child and 45 families feel both children are equal. If two families are selected at random out of 100 families, find the probability distribution of the number of families feel both children are equal.

Q. 23. If a machine is correctly set up, it produces 90% acceptable items. If it is incorrectly set up, it produces only 40% acceptable items. Past experience shows that 80% of the set ups are correctly done. If the machine produces 2 acceptable items, find the probability that the machine is correctly set up.

SECTION – D

Q. 24. Using the method of integration, find the area of the region bounded by the lines: $2x + y = 4$, $3x - 2y = 6$ & $x - 3y + 5 = 0$.

OR

Find the area of the region bonded by the curves $y = \sqrt{5 - x^2}$, $y = |x - 1|$ and x – *axis*.

Q. 25. The sum of length of hypotenuse and a side of a triangle is given. Show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$.

OR

The sum of the perimeter of a circle and a square is 'k' unit, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.

Q. 26. Evaluate : $\int_{-1}^{3/2} |x \sin(\pi x)|\,dx$ OR Evaluate: $\int_0^{\pi} \frac{x}{(a^2cos^2x + b^2sin^2x)}\,dx$.

Q. 27. Show that the lines $\frac{x - a + d}{\alpha - \delta} = \frac{y - a}{\alpha} = \frac{z - a - d}{\alpha + \delta}$ and $\frac{x - b + c}{\beta - \gamma} = \frac{y - b}{\beta} = \frac{z - b - c}{\beta + \gamma}$ are coplanar. Also find the equation of plane containing the lines.

Q. 28. Using E – Transformations find inverse of the matrix $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

Q. 29. A toy company manufactures two type of dolls, A and B. Market tests and available resources have indicated that the combine production level should not exceed 1200 dolls / week and the demand for the dolls of type B is at most half of that for the dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes the profit of `12 and `16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximize the profit .

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