

TEST PAPER – 3
Mathematics – XII

Time : 3 hr

Max Marks : 100

GENERAL INSTRUCTIONS :-

1. All questions are compulsory.
2. SECTION – A comprises of 4 questions of one marks each.
3. SECTION – B comprises of 8 questions of two marks each.
4. SECTION – C comprises of 11 questions of four marks each.
5. SECTION – D comprises of 6 questions of six marks each.
6. Internal choice has been provided in 03 questions of four marks each and 03 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

SECTION – A

- Q. 1. Write the all possible equivalence relation(s) R on set $A = \{1, 2, 3\}$, containing (1, 2) .
- Q. 2. If 'f' and 'g' are two injective functions then 'gof' is injective, but the converse need not be true. Justify your answer with a suitable example.
- Q. 3. Find the angle between the two unit vectors \vec{a} and \vec{b} such that $\vec{a} + \vec{b}$ is also a unit vector.
- Q. 4. If A is a square matrix of order 3 and $|adj(A)| = 144$, then find the value of $|3A|$

SECTION – B

- Q. 5. Find the non principle range for $\operatorname{cosec}^{-1} x$, $|x| \leq 2$
- Q. 6. If foot of perpendicular from (4, -1, 2) on a plane is (-10, 5, 4), find the equation of such plane.
- Q. 7. If $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then prove that $\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$
- Q. 8. If $\vec{a} = i + j + k$, $\vec{b} = i$ and $\vec{c} = xi - j + k$ are coplanar find the value of x
- Q. 9. Evaluate: $_{-1} \int_2 \frac{dx}{x\sqrt{x^2-1}}$
- Q. 10. Find the equation of the curve passing through the point (-2, 3), given that the slope of the tangent to the curve at any point (x, y) is $\frac{2x}{y^2}$.
- Q. 11. For what value of m, the line $y = mx + 1$ is a tangent to the curve $y^2 = 4x$
- Q. 12. In a hurdle race, a player has to cross 10 hurdles. The probability that he clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles?

SECTION – C

- Q. 13. If two equal sides of an isosceles triangle with fixed base 'b' are decreasing at the rate of 3 cm/s. How fast is the area of the triangle decreasing, when the two equal sides are equal to the base.

OR

Using differentials find the approximate value of $\left(\frac{17}{81}\right)^{\frac{1}{4}}$

- Q. 14. Evaluate: $\int \frac{x^x(x^{2x}+1)(\log x + 1)dx}{(x^{4x}+1)}$

- Q. 15. Solve the differential equation : $\sqrt{1 + x^2 + y^2 + x^2y^2} + xy \frac{dy}{dx} = 0$.

- Q. 16. Decompose the vector $5i - 2j + 5k$ into vectors which are parallel & perpendicular to $3i + k$.

OR

If A, B, C, D are four points in space, prove that, $|\vec{AB} \times \vec{CD} + \vec{BC} \times \vec{AD} + \vec{CA} \times \vec{BD}| = 4(\text{area } \Delta ABC)$

P.T.O

Q. 17. Find the values of 'k' so that the function $f(x) = \begin{cases} \frac{\tan^3 x - \tan x}{\cos(x + \frac{\pi}{4})} & ; x \neq \frac{\pi}{4} \\ k & ; x = \frac{\pi}{4} \end{cases}$ is continuous at $x = \frac{\pi}{4}$.

Q. 18. Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line $\vec{r} = 2i - j + 2k + \lambda(3i + 4j + 2k)$ and the plane $\vec{r} \cdot (i - j + k) = 5$.

OR

Find the image of the point $(1, 3, 4)$ in the plane $2x - y + z + 3 = 0$.

Q. 19. In a group of 100 families, 30 families like male child, 25 families like female child and 45 families feel both children are equal. If two families are selected at random out of 100 families, find the probability distribution of the number of families feel both children are equal.

What is the importance in the society to develop the feeling that both children are equal?

Q. 20. Two third of the students in a class are sincere about their study and rest are careless Probability of passing in examination are 0.7 and 0.2 for sincere and careless students respectively, A Student is chosen and is found to be passed what is the probability that he/she was sincere.

Explain the importance of sincerity for a student.

Q. 21. Find the maximum area of the isosceles triangle inscribed in a circle of radius r .

Q. 22. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find the value of 'k' so that $A^2 = kA - I$.

Q. 23. If $y = \cot^{-1} \left(\frac{x^x - x^{-x}}{2} \right)$; prove that $\frac{dy}{dx} \Big|_{x=1} = -1$.

SECTION - D

Q. 24. Using the method of integration, find the area of the region bounded by the lines: $2x + y = 4$, $3x - 2y = 6$ & $x - 3y + 5 = 0$.

OR

Find the area of the region enclosed between the two circles $x^2 + y^2 = 1$ and $(x - 1)^2 + y^2 = 1$

Q. 25. Let $A = \{1, 2, 3, \dots, 9\}$ and R be the relation in $A \times A$ defined by $(a, b) R (c, d)$ if $a + d = b + c$, for $(a, b), (c, d) \in A \times A$. Prove that R is an equivalence relation. Also obtain the equivalence class $[(2, 5)]$.

OR

If the function $f: W \rightarrow W$ defined by $f(x) = \begin{cases} x - 1 & ; x \text{ is odd} \\ x + 1 & ; x \text{ is even} \end{cases}$ Show that f is invertible. Find the inverse of f . Here W is the set of all whole numbers.

Q. 26. If the lines $\vec{r} = i + j + \lambda(i + 2j - k)$ and $\vec{r} = i + j + \mu(-i + j - 2k)$ lies in a plane. Then find the distance of the plane, containing the lines, from origin.

Q. 27. Evaluate: $\int_0^{\pi/2} \frac{dx}{(a^2 \cos^2 x + b^2 \sin^2 x)^2}$

Q. 28. If $x \neq y \neq z$, and $D = \begin{vmatrix} x & x^2 & 1 + x^3 \\ y & y^2 & 1 + y^3 \\ z & z^2 & 1 + z^3 \end{vmatrix} = 0$, then show that $1 + xyz = 0$.

Q. 29. A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 gm) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of same quantity of food Q contain 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at least 300 units of cholesterol. How many packets of each food should be used to minimize the amount of vitamin A in the diet? What is the minimum amount of vitamin A in the diet? *Explain the importance of balanced diet for a student.*

OR

Two tailors, A and B, earn Rs 150 and Rs 200 per day, respectively. A can stitch 6 shirts and 4 pants, while B can stitch 10 shirts and 4 pants per day. Form a L.P.P to minimize the labour cost to produce at least 60 shirts and 32 pants and solve it.
