6 – Marks

(1) Using E – transformations find inverse of the matrix $\mathbf{A} = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

(2) Solve the system of linear equations using matrix method , $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$

$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$, $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

(3) The sum of the perimeter of a circle and a square is **'*k*'** unit ,where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle**.**

(4) The sum of length of hypotenuse and a side of a triangle is given**.** Show that the area of the triangle is maximum when the angle between them is $\pi / 3$.

(5) Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$ **.**

(6) **(i)** Find the intervals in which the function ***'f '*** given by $f(x) = \frac{4\sin x - 2x - x\cos x}{2 + \cos x}$ is increasing & decreasing**.**

(ii) If two equal sides of an isosceles triangle with fixed base **'b'** are decreasing at the rate of **3cm/s.** How fast is the area of the triangle decreasing ,when the two equal sides are equal to the base**.**

4 – Marks

(7) Let $\mathbf{A} = \mathbf{N} \times \mathbf{N}$ and $*$ be the binary operation on **A** defined by $(a , b) * (c , d) = (a + c , b + d)$**.** Show that '$*$' is **commutative** & **associative.** Find the identity element for '$*$' on **A** if any.

(8) Show that the function $f : \mathbf{R}_+ \rightarrow [-5 \;\; \infty)$ defined by $f(x) = 9x^2 + 6x - 5$ is invertible**.** Also find f^{-1} **.**

(9) Let $\mathbf{A} = \mathbf{R} - \{2\}$ and $\mathbf{B} = \mathbf{R} - \{3\}$**.** Consider the function $f : \mathbf{A} \rightarrow \mathbf{B}$ defined by $f(x) = \frac{3x + 1}{x - 2}$ **.**

Show that '*f*' is bijective**,** with inverse $f^{-1}(x) = \frac{2x + 1}{x - 3}$

(10) Show that the relation R in the set **A = { x : x is an integer & 0 ≤ x ≤12 }** as **:** **R = { (a , b) : | a – b | = 4k }** with **' k'** an integer**,** is an equivalence relation.

(11) Prove that **:** $2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{5\sqrt{2}}{7} + 2\tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

(12) Prove that **:** $\tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} + \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} = \frac{2b}{a}$

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(13) Prove that, $sin^{-1}\left(\frac{4}{5}\right) + sin^{-1}\left(\frac{5}{13}\right) + sin^{-1}\left(\frac{16}{65}\right) = \frac{\pi}{2}$

(14) If , $tan^{-1}\left(\frac{x-1}{x-2}\right) + tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ **.** Find the value of x **.**

(15) Find the matrix **' X '** so that $X . \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

(16) If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $f(x) = x^2 - 5x - 5$, then show that $f(A) = O$. Hence find A^{-1} .

(17) Using properties of determinant prove that $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$

(18) If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$, are the p^{th}, q^{th} and r^{th} terms of an **A.P** then prove that $\begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = 0$

(19) Find **'*k*'** if the function
$$f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & ; \ x < 0 \\ k & ; \ x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}} - 4} & ; \ x > 0 \end{cases}$$
, is continuous at **x = 0.**

(20) If $x = a(\cos t + t \sin t)$ & $y = a(\sin t - t \cos t)$; find $\frac{d^2y}{dx^2}$

(21) If $(x-a)^2 + (y-b)^2 = c^2$ for some **c** > **0,**

then prove that $\frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant independent of **'*a*'** and **'*b*'.**

(22) Find $\frac{dy}{dx}$ *if* , $y = \sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$..

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