BACK – UP TEST – 1 B

Mathematics – XII

Max Marks – 100
Time – 3 hr

6 – Marks

(1) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & -3 \\ -3 & 2 & -4 \end{bmatrix}$, find A^{-1}.

Hence solve the following system of equations: $x + 2y - 3z = -4$, $2x + 3y + 2z = 2$, $3x - 3y - 4z = 11$

(2) A school wants to award its students for the values of Honesty, Regularity and Hard work with a total cash award d of **Rs. 6000.** Three times the award money for Hard Work added to that given for Honesty amounts to **Rs 11000.** The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award money for value, using matrix method. Apart from these values, namely Honesty, Hard work and Regularity, suggest one more value which the school must include for award.

(3) If length of three sides of a trapezium other than base are equal to **10 cm,** then find the area of the trapezium when it is maximum.

(4) A given quantity of metal is to be cast into a half cylinder (a rectangular base and semi – circular ends). Show that the total surface area will be least when the ratio of the length of the cylinder to the diameter of the ends is $\pi : (\pi + 2)$.

(5) A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is $2\ m$ and volume is $8\ m^3$. If building of tank costs **Rs 70** / m^2 for the base and **Rs 45** / m^2 for sides. What is the cost of least expensive tank?

(6) **(i)** Find the equation of normal to the curve $x^2 = 4y$ which passes through the point **(1 , 2)** .

(ii) Find the approximate value of $(3.968)^{3/2}$.

4 – Marks

(7) Define a binary operation $*$ on the set {0,1,2,3,4,5} as: $a * b = a + b \quad ; a + b < 6$
$= a + b - 6 \quad ; a + b \geq 6$

Show that **'0'** is the **identity** of this operation & each element **'*a*'** of the set is invertible with **'6 – *a*'** being the inverse of ***a*** .

(8) Let $f : \mathbf{N} \rightarrow \mathbf{R}$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : \mathbf{N} \rightarrow \mathbf{S}$, where **S** is the range of f, is invertible. Find the inverse of f .

(9) Consider the function $f : \mathbf{N} \rightarrow \mathbf{N}$ defined by

$$f(n) = \begin{cases} \dfrac{n+1}{2} & ; \text{if } n \text{ is odd} \\ \dfrac{n}{2} & ; \text{if } n \text{ is even} \end{cases}$$

Is f injective and surjective?

(10) Let N br the set of all natural numbers and R be the relation on **N × N** defined by **:** $(a, b)\ \mathrm{R}\ (c, d) \Leftrightarrow ad = bc$. Show that relation R is an equivalence relation.

(11) Prove that : $\cos\left[\tan^{-1}\{\sin(\cot^{-1} x)\}\right] = \dfrac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$

(12) If, $y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$, prove that $\sin y = \tan^2\left(\dfrac{x}{2}\right)$.

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(13) Prove that : $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) = \pi$.

(14) Find the value of x if , $\sin^{-1}(1 - x) - 2\sin^{-1} x = \frac{\pi}{2}$.

(15) Find the matrix **'D'** if $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 5 \\ 3 & 5 \end{bmatrix}$ satisfies **CD – AB = O.**

(16) For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the numbers **'*a*'** and **'*b*'** such that $A^2 + aA + bI = O$, also find A^{-1}.

(17) Using properties of determinant prove that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is always negative, if $a \neq b \neq c > 0$

(18) If x , y & z are all positive and 10^{th}, 15^{th}, and 25^{th} term of a **G.P** then prove that, $\begin{vmatrix} \log x & 2 & 1 \\ \log y & 3 & 1 \\ \log z & 5 & 1 \end{vmatrix} = 0$

(19) Find the values of **'*k*'** so that the function $f(x) = \begin{cases} \dfrac{\sqrt{2}\cos x - 1}{\cot x - 1} & ; x \neq \dfrac{\pi}{4} \\ k & ; x = \dfrac{\pi}{4} \end{cases}$ is continuous at $x = \frac{\pi}{4}$

(20) If, $x\sqrt{1 + y} + y\sqrt{1 + x} = 0$; $-1 < x < 1$, then prove that $\frac{dy}{dx} = -\frac{1}{(1 + x)^2}$

(21) If $y = \sin^{-1}\left(\frac{\sqrt{1 + x} - \sqrt{1 - x}}{2}\right)$; then prove that $\frac{dy}{dx} = \frac{1}{2\sqrt{1 - x^2}}$

(22) Find $\frac{dy}{dx}$ if , $y = x^{\sin x} + \{\sin x\}^{\cos x}$.

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