

6 – Marks

- (1) There are two type of fertilizers  $F_1$  &  $F_2$ .  $F_1$  consists of 10 % nitrogen and 6 % phosphoric acid and  $F_2$  consists of 5% nitrogen & 10% phosphoric acid. After testing the soil condition, a farmer finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If  $F_1$  costs Rs 6 / kg and  $F_2$  costs Rs 5 / kg, determine how much of each type of fertilizers should be used so that the nutrient requirement are met at a minimum cost. What is the minimum cost.
- (2) An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline . What is the maximum profit ?
- (3) Find the area of region  $\{ (x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2 \}$
- (4) Using the method of integration find the area of the region bounded by the lines:  $2x + y = 4$ ,  $3x - 2y = 6$  &  $x - 3y + 5 = 0$ .
- (5) Solve the initial value problem :  $\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x$  ;  $y(\pi/2) = 0$
- (6) A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively  $\frac{3}{10}$ ,  $\frac{1}{5}$ ,  $\frac{1}{10}$  and  $\frac{2}{5}$ . The probabilities that he will be late are  $\frac{1}{4}$ ,  $\frac{1}{3}$  and  $\frac{1}{12}$  if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train?
- (7) A man is to known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six .
- (8) Evaluate :  $\int_0^{\pi} x dx / (a^2 \cos^2 x + b^2 \sin^2 x)$
- (9) A variable plane which always remains at a constant distance '3p' from origin, cuts the coordinate axes at A, B, C respectively. Prove that the locus of the centroid of the triangle ABC is  $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$ .
- (10) Find the equation of the plane through the line of intersection of the planes  $x + y + z = 1$  and  $2x + 3y + 4z = 5$ , which is perpendicular to the plane  $x - y + z = 0$ .

4 – Marks

- (11) If 3 defective bulbs are accidentally mixed with 7 good ones, and three bulbs are drawn successively. What is the average number of defective bulbs drawn ?
- (12) A and B throws a die alternately till one of them gets a '6' and wins the game. Find their respective probabilities of winning, if A starts the game.
- (13) If  $\alpha, \beta, \gamma$  and  $\delta$  are the angles made by any line with the four diagonals of any cube, then prove that :  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3}$
- (14) If with reference to the right handed system of mutually perpendicular unit vectors  $i, j, k$ ;  
 $\vec{a} = 3i - j, \vec{b} = 2i + j - 3k$ , then express  $\vec{b}$  in the form  $\vec{b} = \vec{b}_1 + \vec{b}_2$ , where  $\vec{b}_1$  is parallel to  $\vec{a}$  and  $\vec{b}_2$  is perpendicular to  $\vec{a}$
- (15) Let  $\vec{a} = i + 4j + 2k, \vec{b} = 3i - 2j + 7k, \vec{c} = 2i - j + 4k$ . Find a vector  $\vec{d}$  which is perpendicular to both  $\vec{a}$  and  $\vec{b}$ , &  $\vec{c} \cdot \vec{d} = 15$ .
- (16) Show that the vectors  $\vec{a}, \vec{b}$  and  $\vec{c}$  coplanar if  $\vec{a} + \vec{b}, \vec{b} + \vec{c}$  and  $\vec{c} + \vec{a}$  coplanar.
- (17) Show that the general solution of the differential equation  $(x^2 + x + 1) dy + (y^2 + y + 1) dx = 0$  is given by  $(x + y + 1) = A(1 - x - y - 2xy)$
- (18) Evaluate :  $\int \{ \sqrt{\tan x} + \sqrt{\cot x} \} dx$
- (19) Evaluate :  $\int \left\{ \log(\log x) + \frac{1}{(\log x)^2} \right\} dx$
- (20) Evaluate :  $\int \frac{(\sin \sqrt{x} - \cos \sqrt{x}) dx}{(\sin \sqrt{x} + \cos \sqrt{x})}$

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