

6 – Marks

- (1) A manufacturer makes cycles and scooters. Processing of these products is done on two machines **A** and **B**. The cycle parts need **1** hours on machine **A** and **3** hours on machine **B**. Parts of a scooter needs **3** hours on machine **A** and **1** hours on machine **B**. Both machines are available for **12** hours per day. Profit gained by the manufacturers from a cycle and a scooter is **Rs.1000** and **Rs.3000** respectively. Find with the help of a graph what should be the daily production of each of the two products to maximize the profit? Which of the above modes of transport is a better option and why?
- (2) A toy company manufactures two type of dolls, **A** and **B**. Market tests and available resources have indicated that the combine production level should not exceed **1200** dolls / week and the demand for the dolls of type **B** is at most half of that for the dolls of type **A**. Further , the production level of dolls of type **A** can exceed three times the production of dolls of other type by at most **600** units. If the company makes the profit of **Rs12** and **Rs16** per doll respectively on dolls **A** and **B**, how many of each should be produced weekly in order to maximize the profit .
- (3) Find the area lying above x-axis, included between the circle $x^2 + y^2 = 8x$ and interior to parabola $y^2 = 4x$.
- (4) Using the method of integration find the area of the region bounded by the curves $y^2 = x$ and $x + y = 2$.
- (5) Solve the initial value problem : $x dy + y dx - x dx + xy \cot x dx = 0$; $x = 0, y = 1$
- (6) In a test, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct, given that he copied it, is $\frac{1}{8}$. Find the probability that he knew the answer to the question, given that he correctly answered it. Copying is akin to using a painkiller. Explain the analogy.
- (7) Suppose a girl throws a die. If she gets **5** or **6** she tosses a coin three times and notes the number of heads. If she gets **1, 2, 3** or **4**, she tosses the coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw **1, 2, 3** or **4** with the die ?
- (8) Evaluate : $-\int_1^{3/2} |x \sin \pi x| dx$
- (9) Find the distance of the point **(2, 3, 4)** from the plane $3x + 2y + 2z + 5 = 0$, measured parallel to the line $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$.
- (10) Find the cartesian as well as the vector equation of the plane through the intersection of the planes $\vec{r} \cdot (2\mathbf{i} + 6\mathbf{j}) + 12 = 0$ and $\vec{r} \cdot (3\mathbf{i} - \mathbf{j} + 4\mathbf{k}) = 0$, and at a unit distance from origin .

4 – Marks

- (11) A coin is biased so that the head is three times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.
- (12) A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.
- (13) Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line $\vec{r} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \lambda(3\mathbf{i} + 4\mathbf{j} + 2\mathbf{k})$ and the plane $\vec{r} \cdot (\mathbf{i} - \mathbf{j} + \mathbf{k}) = 5$.
- (14) If $\vec{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\vec{b} = \mathbf{j} - \mathbf{k}$, find a vector \vec{c} such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$.
- (15) If a unit vector \vec{a} makes angles $\frac{\pi}{3}$ with \mathbf{i} , $\frac{\pi}{4}$ with \mathbf{j} and an acute angle θ with \mathbf{k} , then find θ and hence the vector \vec{a} .
- (16) If the vectors $\vec{a} = x\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\vec{b} = \mathbf{i} + y\mathbf{j} + \mathbf{k}$ and $\vec{c} = \mathbf{i} + \mathbf{j} + z\mathbf{k}$ are coplanar, then prove that $\frac{1}{1-x} + \frac{1}{1-y} + \frac{1}{1-z} = 1$, where $x, y, z \neq 1$.
- (17) Solve the initial value problem : $(x - y)(dx + dy) = dx - dy$; $y = -1, x = 0$
- (18) Evaluate : $\int \sqrt{\tan x} dx$
- (19) Evaluate : $\int_0^1 \log(1+x) dx / (1+x^2)$
- (20) Evaluate : $\int dx / \sqrt{\sin^3 x \sin(x + \alpha)}$
