PUSH – UP TEST – 1

Mathematics – XI

Max Marks – 100
Time – 3 hr

4 – Marks

(1) Show that , $\cot x \cdot \cot 2x - \cot 2x \cdot \cot 3x - \cot 3x \cdot \cot x = 1$

(2) Find the value of $\tan\left(\frac{\pi}{8}\right)$.

(3) Show that , $\cos^2 x + \cos^2\left\{x + \frac{\pi}{3}\right\} + \cos^2\left\{x - \frac{\pi}{3}\right\} = \frac{3}{2}$

(4) Find the general solution of the equation ; $\sin 2x - \sin 4x + \sin 6x = 0$.

(5) In a triangle **ABC** prove that : $\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$

(6) Show that : $\frac{\sec 8x - 1}{\sec 4x - 1} = \frac{\tan 8x}{\tan 2x}$

(7) Using Principle of Mathematical Induction prove that,

for all $n \geq 1$, $1^2 + 2^2 + 3^2 + \cdots + n^2 > \frac{n^3}{3}$

(8) Using Principle of Mathematical Induction prove that :

for all $n \geq 1$, $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \cdots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$

(9) Using Principle of Mathematical Induction prove that :

for all $n \geq 1$, $1.3 + 3.5 + 5.7 + \cdots + (2n-1)(2n+1) = \frac{n(4n^2 + 6n - 1)}{3}$

(10) Using Principle of Mathematical Induction prove that :
for all $n \geq 1$, $n^3 + (n+1)^3 + (n+2)^3$ is divisible by **9.**

(11) Using Principle of Mathematical Induction prove that :

for all $n \geq 1$, $1 + \frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \cdots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{(6n+4)}$

(12) Using Principle of Mathematical Induction prove that,
for all $n \geq 1$, $2.7^n + 3.5^n - 5$ is divisible by **24.**

(13) If 'α' and 'β' are two different complex numbers with $|\beta| = 1$, then find $\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right|$

(14) If $x - iy = \sqrt{\frac{a - ib}{c - id}}$, then prove that $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$

(15) Find the real numbers 'x' & 'y' if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$.

(16) If $(x + iy)^3 = u + iv$, then show that $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$

P.T.O

6 – Marks

(17) In a triangle **ABC** prove that : $(b^2 - c^2)\cot A + (c^2 - a^2)\cot B + (a^2 - b^2)\cot C = 0$

(18) Convert the complex number , $\dfrac{i - 1}{\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)}$, into polar form

(19) Solve for 'x' : $x^2 - 5x + i\,x - i + 18 = 0.$

(20) Find the square root of the complex number $8 - 15i$

(21) Using Principle of Mathematical Induction prove that :

for all $n \geq 1$, $\cos x + \cos 2x + \cos 3x + \text{- - -} + \cos nx = \sin\left(\frac{nx}{2}\right).\operatorname{cosec}\left(\frac{x}{2}\right).\cos\left(\frac{(n+1)\,x}{2}\right)$

(22) Find the value of $\sin\left(\frac{x}{2}\right)$, $\cos\left(\frac{x}{2}\right)$ and $\tan\left(\frac{x}{2}\right)$ if $\sin x = -\frac{1}{4}$; with $\pi < x < \frac{3\pi}{2}$

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