PUSH – UP TEST – 3

Mathematics – XI

Max Marks – 100
Time – 3 hr

4 – Marks

(1) The sums of n terms of two arithmetic progressions are in the ratio $5n + 4 : 9n + 6$. Find the ratio of their 18^{th} terms.

(2) If the first and the n^{th} term of a **G.P.** are 'a' and 'b', respectively, and if '**P**' is the product of n terms, prove that ; $P^2 = (ab)^n$.

(3) Let S be the sum, P the product and R the sum of reciprocals of n terms in a ***G.P.*** Prove that $P^2R^n = S^n$.

(4) Find the sum to n terms of the series: **0.6 + 0.66 + 0.666 + 0.6666 + - - - .**

(5) The sum of three numbers in **G.P.** is **56.** If we subtract **1, 7, 21** from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

(6) Find the sum to n terms of the series : **5 + 11 + 19 + 29 + 41 - - - .**

(7) If a, b, c, d are in **G.P,** prove that $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in **G.P.**

(8) If p, q, r are in **G.P.** and the equations, $px^2 + 2qx + r = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then show that $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in **A.P**

(9) Assuming that straight lines work as the plane mirror for a point, find the image of the point **(1, 2)** in the line $x - 3y + 4 = 0$.

(10) If '**p**' and '**q**' are the lengths of perpendiculars from the origin to the lines $x\cos\theta - y\sin\theta = k\cos 2\theta$ and $x\sec\theta + y\,\text{cosec}\,\theta = k$, respectively, prove that $p^2 + 4q^2 = k^2$.

(11) A ray of light passing through the point **(1 , 2)** reflects on the x – axis a the point A and the reflected ray passes through point **(5 , 3),** then find the coordinate of the point A.

(12) Find perpendicular distance from the origin of the line joining the points $(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$.

(13) If three lines $y = m_1.x + c_1$; $y = m_2.x + c_2$ and $y = m_3.x + c_3$ are concurrent, prove that $m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$.

(14) Find the equation of the line passing through the point **(2 , 2)** and cutting off intercepts on axes whose sum is **9.**

(15) A person standing at the junction (crossing) of two straight paths represented by the equations $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ wants to reach the path whose equation is $6x - 7y + 8 = 0$ in the least time. Find equation of the path that he should follow.

(16) Prove that the product of the lengths of the perpendiculars drawn from the points $(\sqrt{a^2 - b^2}, 0)$ and $(-\sqrt{a^2 - b^2}, 0)$ to the line $\frac{x\cos\phi}{a} + \frac{y\sin\phi}{b} = 1$ is b^2.

P.T.O

6 – Marks

(17) Show that: $\dfrac{1 \times 2^2 + 2 \times 3^2 + --- + n \times (n+1)^2}{1^2 \times 2 + 2^2 \times 3 + --- + n^2 \times (n+1)} = \dfrac{3n+5}{3n+1}$

(18) If ***a*** and ***b*** are the roots of $x^2 - 3x + p = 0$ and ***c, d*** are roots of $x^2 - 12x + q = 0$, where ***a, b, c, d*** form **G.P.** Prove that $(q + p) : (q - p) = 17:15$.

(19) Find the sum of the series up to ***n*** terms: $\dfrac{1^3}{1} + \dfrac{1^3 + 2^3}{1+3} + \dfrac{1^3 + 2^3 + 3^3}{1+3+5} + - - -$

(20) A line is such that its segment between the lines $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point **(1, 5).** Obtain its equation.

(21) Find the distance of the line $4x - y = 0$ from the point **P (4, 1)** measured along the line making an angle of **135°** with the positive x-axis.

(22) Find the direction in which a line must be drawn through the point **(–1 , 2)** so that its point of intersection with the line $x + y = 4$ may be at a distance **3 units** from this point **.**

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