PUSH – UP TEST – 1 | Mathematics – XII | Max Marks – 100, Time – 3 hr

4 – Marks

(1) Find **'x'** if, $\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$.

(2) Show that : $\tan^{-1}(1/5) + \tan^{-1}(1/7) + \tan^{-1}(1/3) + \tan^{-1}(1/8) = \pi/4$.

(3) If, $\cos^{-1}(x/a) + \cos^{-1}(y/b) = \alpha$ Prove that $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\alpha + \frac{y^2}{b^2} = \sin^2\alpha$

(4) Show that : $\cot^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\} = \frac{x}{2}$, $x \in [0 \quad \pi/2]$

(5) Solve the equation, $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{\tan^{-1}x}{2}$; $x > 0$.

(6) Prove that : $\sin^{-1}(12/13) + \cos^{-1}(4/5) + \tan^{-1}(63/16) = \pi$.

(7) Show that : $\tan^{-1}\left\{\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right\} = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$, $x \in [0 \quad \pi/2]$

(8) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, Prove that $x^2 + y^2 + z^2 + 2xyz = 1$.

(9) If, $A = \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}$, then by using **P.M.I**, show that, $A^n = \begin{pmatrix} \cos nx & \sin nx \\ -\sin nx & \cos nx \end{pmatrix}$, $\forall\, n \in N$.

(10) If $A = \begin{bmatrix} 0 & -\tan(x/2) \\ \tan(x/2) & 0 \end{bmatrix}$, Show that, $(I + A) = (I - A)\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$

(11) Express the matrix, $A = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$, as sum of a symmetric and a skew – symmetric matrices.

(12) For the matrix $A = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$, find the numbers **'a'** and **'b'** such that $A^2 + aA + bI = O$, also find A^{-1}.

(13) If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ & $B = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix}$, then verify $(AB)^T = B^T . A^T$.

(14) Find the matrix **'D'** if $A = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 2 \\ 7 & 4 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 5 \\ 3 & 5 \end{pmatrix}$ satisfies $CD - AB = O$.

(15) Find the value of **'x'** if $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{pmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ x \end{pmatrix} = O$.

(16) Prove by using properties of determinant $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc + ab + bc + ac$

P.T.O

6 – Marks

(17) Using elementary transformations find inverse of the matrix $\begin{pmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{pmatrix}$

Using properties of determinant prove the following (18 to 21):

(18) $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3.$

(19) $\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x).$

(20) $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3.$

(21) $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2.$

(22) Use the product $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix}\begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix}$,

to solve the system of linear equations : $x - y + 2z = 1$,
$2y - 3z = 1$,
$3x - 2y + 4z = 2$, by matrix method .

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