Mathematics – XII

4 – Marks

(1) Discuss the continuity of the function $f(x) = [x]$, where [ ] is the greatest integer function, defined as an integer less and equals to x.

(2) If $y = e^{a\cos^{-1}x}$; $-1 < x < 1$, show that, $(1 - x^2).y_2 - x.y_1 - a^2 y = 0$.

(3) If $y^x + x^y + x^x = a^b$, find the value of $\frac{dy}{dx}$.

(4) Prove that $\frac{d}{dx}\left\{\frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right\} = \sqrt{a^2 - x^2}$

(5) If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, find dy / dx.

(6) If $x = a(\cos t + t.\sin t)$ & $y = a.(\sin t - t.\cos t)$; prove that $d^2y / dx^2 = \sec^3 t / at$.

(7) If $y = 3\cos(\log x) + 5\sin(\log x)$, prove that, $x^2 y_2 + x y_1 + y = 0$.

(8) If $y = a^{t + 1/t}$; $x = (t + 1/t)^a$ then find dy / dx

(9) If $x = \sqrt{a^{\sin^{-1}t}}$ and $y = \sqrt{a^{\cos^{-1}t}}$ prove that $\frac{dy}{dx} = -\frac{y}{x}$.

(10) If $y = x^{\sin x} + (\sin x)^{\cos x}$. Find $\frac{dy}{dx}$

(11) If $x^2 + y^2 = t - \frac{1}{t}$; $x^4 + y^4 = t^2 + \frac{1}{t^2}$; prove that ; $\frac{dy}{dx} = \frac{1}{x^3 y}$.

(12) If $y = \tan^{-1}\left(\frac{\sqrt{1 + x^2} - 1}{x}\right)$; $\frac{dy}{dx} = \frac{1}{2(1 + x^2)}$

(13) If $y = \cot^{-1}\left(\frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}\right)$; $0 < x < \pi / 2$, then prove that $dy / dx = 1 / 2$.

(14) Verify the Mean Value Theorem for the function, $f(x) = x^3 - 5x^2 - 3x$; $x \in [1 \quad 3]$.

(15) If two equal sides of an isosceles triangle with fixed base **'b'** are decreasing at the rate of **3cm/s.** How fast is the area of the triangle decreasing ,when the two equal sides are equal to the base**.**

(16) Show that $y = \log(1 + x) - \frac{2x}{2 + x}$; $x > -1$; is an increasing function of x throughout its domain .

(17) Find the point on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with the axes.

(18) Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is perpendicular to the line $5y - 15x = 13$.

(19) Find the intervals in which the function $f(x) = (x + 1)^3.(x - 3)^3$, is
(i) strictly increasing & **(ii)** strictly decreasing

P.T.O

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6 – Marks

(20) The sum of the perimeter of a circle and a square is **'k'** unit ,where k is some constant .Prove that the sum of their areas is least when the side of square is double the radius of the circle**.**

(21) Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius **R** is **2R / √3.** Also find the maximum volume **.**

(22) If length of three sides of a trapezium other than base are equal to **10cm,** then find the area of the trapezium when it is maximum.

(23) A rectangle is inscribed in a semi – circle of radius **'R'** with one of its side on the diameter of the semi **–** circle**.** Find the dimension of the rectangle so that the rectangle has maximum area**.**

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