PUSH – UP TEST – 4

Mathematics – XII

Max Marks – 100
Time – 3 hr

4 – Marks

(1) Solve the equation : $(\tan^{-1}y - x)\,dy = (1 + y^2)\,dx$

(2) Solve the equation : $\{x\cos(y/x) + y\sin(y/x)\}\,y\,dx = \{y\sin(y/x) - x\cos(y/x)\}\,x\,dy$

(3) Show that the general solution of the differential equation $(x^2 + x + 1)\,dy + (y^2 + y + 1)\,dx = 0$ is given by $(x + y + 1) = A(1 - x - y - 2xy)$

(4) Three vectors $\vec{a}, \vec{b}, \vec{c}$ satisfy the condition $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Evaluate the quantity $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$, if $|\vec{a}| = 1$, $|\vec{b}| = 4$, $|\vec{c}| = 2$.

(5) If a unit vector $\vec{a}$ makes angles $\pi/3$ with $\mathbf{i}$, $\pi/4$ with $\mathbf{j}$ and an acute angle θ with $\mathbf{k}$, then find θ and hence the vector $\vec{a}$.

(6) If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitude, find the angles made by the vector $\vec{a} + \vec{b} + \vec{c}$ with $\vec{a}, \vec{b}, \vec{c}$ respectively. Hence show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$.

(7) Let $\vec{a} = \mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$, $\vec{b} = 3\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$, $\vec{c} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, & $\vec{c}\cdot\vec{d} = 15$.

(8) Find a unit vector perpendicular to each of the vector $\vec{a} + \vec{b}$ & $\vec{a} - \vec{b}$, where $\vec{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\vec{b} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.

(9) Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and each one of them being perpendicular to the sum of the two, find $|\vec{a} + \vec{b} + \vec{c}|$.

(10) If with reference to the right handed system of mutually perpendicular unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$; $\vec{a} = 3\mathbf{i} - \mathbf{j}$, $\vec{b} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$, then express $\vec{b}$ in the form $\vec{b} = \vec{b}_1 + \vec{b}_2$, where $\vec{b}_1$ is parallel to $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$

(11) If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors, such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, and $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, then prove that angle between $\vec{a}$ and $\vec{b}$ is 60°.

(12) Find the value of **'p'** so that the lines: $\dfrac{1 - x}{3} = \dfrac{7y - 14}{2p} = \dfrac{z - 3}{2}$; $\dfrac{7 - 7x}{3p} = \dfrac{y - 5}{1} = \dfrac{6 - z}{5}$, are perpendicular.

(13) If α, β, γ and δ are the angles made by any line with the four diagonals of any cube, then prove that : $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$

P.T.O

6 – Marks

(14) Solve the initial value problem : $(1 + e^{2x})dy + (1 + y^2)\, e^{x}\, dx = 0$; $y(0) = 1$

(15) Solve the initial value problem : $y^{1} + y \cot x = 2x + x^2 \cot x$; $y(\pi/2) = 0$

(16) Solve the initial value problem : $(x^3 + x^2 + x + 1)dy = (2x^2 + x)\, dx$; $y(0) = 1$.

(17) A variable plane which always remains at a constant distance **'p'** from origin, cuts the coordinate axes at **A, B, C** respectively. Prove that the locus of the centroid of the triangle **ABC** is $x^{-2} + y^{-2} + z^{-2} = 9p^{-2}$.

(18) Find the equation of the plane passing through the line of intersection of the planes

$\vec{r}\cdot(i + j + k) = 1$, $\vec{r}\cdot(2i + 3j - k) + 4 = 0$, and parallel to the **x – axis .**

(19) Find the equation of the plane passing through the line of intersection of the planes

$\vec{r}\cdot(i + 2j + 3k) = 4$, and $\vec{r}\cdot(2i + j - k) + 5 = 0$, and which is perpendicular to the plane

$\vec{r}\cdot(5i + 3j - 6k) + 8 = 0$.

(20) Find the shortest distance between the lines : $\dfrac{x+1}{7} = \dfrac{y+1}{-6} = \dfrac{z+1}{1}$ and $\dfrac{x-3}{1} = \dfrac{y-5}{-2} = \dfrac{z-7}{1}$

(21) Find the distance between the lines : $\vec{r} = i + 2j - 4k + \lambda(2i + 3j + 6k)$;
$\vec{r} = 3i + 3j - 5k + \mu(2i + 3j + 6k)$.

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