PUSH – UP TEST – 5

Mathematics – XII

Max Marks – 100
Time – 3 hr

6 – Marks

(1) A toy company manufactures two type of dolls**, A** and **B.** Market tests and available resources have indicated that the combine production level should not exceed **1200** dolls / week and the demand for the dolls of type **B** is at most half of that for the dolls of type **A.** Further , the production level of dolls of type **A** can exceed three times the production of dolls of other type by at most **600** units . If the company makes the profit of **Rs12** and **Rs16** per doll respectively on dolls **A** and **B,** how many of each should be produced weekly in order to maximize the profit **.**

(2) There are two type of fertilizers $\mathbf{F_1}$ & $\mathbf{F_2}$**.** $\mathbf{F_1}$ consists of **10** % nitrogen and **6** % phosphoric acid and $\mathbf{F_2}$ consists of **5**% nitrogen & **10**% phosphoric acid**.** After testing the soil condition, a farmer finds that he needs at least **14** kg of nitrogen and **14** kg of phosphoric acid for his crop. If $\mathbf{F_1}$ costs **Rs 6 / kg** and $\mathbf{F_2}$ costs **Rs 5 / kg,** determine how much of each type of fertilizers should be used so that the nutrient requirement are met at a minimum cost**.** What is the minimum cost**.**

(3) A young man rides his motorcycle at **25 km / hr,** he has to spend **Rs 2 / km** on petrol **;** if he rides it at a faster speed of **40 km / hr,** the petrol cost increases to **Rs 5 / km.** He has **Rs 100** to spend on petrol and wishes to find the maximum distance he can travel within **1 hr.** Express this as an **L.P.P.** and solve it.

(4) A factory manufactures two types of screws**, A** and **B,** each type requiring the use of two machines**,** an automatic and a hand operated. It takes **4** minutes on the automatic and **6** minutes on hand operated machines to manufacture a package of screws A**,** while it takes **6** minutes on automatic and **3** minutes on the hand operated machines to manufacture a package of screws **B.** Each machine is available for at the most **4** hours on any day. The manufacturer can sell a package of screws **A** at a profit of **Rs7** and screws **B** at a profit of **Rs10.** Assuming that he can sell all the screws he can manufacture, how many packages of each type should the factory owner produce in a day in order to maximize his profit**?** Determine the maximum profit**.**

(5) An insurance company insured **2000** scooter drivers**, 4000** car drivers and **6000** truck drivers**.** The probability of an accidents are **0.01, 0.03** and **0.15** respectively**.** One of the insured persons meets an accident**.** What is the probability that he is a scooter driver **?**

(6) A laboratory blood test is **99%** effective in detecting a certain disease when it is in fact**,** present**.** However**,** the test also yields a false positive result for **0.5%** of the healthy person tested **(** i.e. if a healthy person is tested, then, with probability **0.005,** the test will imply he has the disease **).** If **0.1** percent of the population actually has the disease**,** what is the probability that a person has the disease given that his test result is positive **?**

(7) Suppose a girl throws a die**.** If she gets **5** or **6** she tosses a coin three times and notes the number of heads**.** If she gets **1, 2, 3** or **4,** she tosses the coin once and notes whether a head or tail is obtained**.** If she obtained exactly one head, what is the probability that she threw **1, 2, 3** or **4** with the die **?**

(8) A man is to known to speak truth **3** out of **4** times**.** He throws a die and reports that it is a six**.** Find the probability that it is actually a six **.**

(9) Let $\mathbf{A = N \times N}$ and $*$ be the binary operation on **A** defined by $\mathbf{(a , b) * (c , d) = (a + c , b + d)}$**.**
Show that **'$*$'** is **commutative** & **associative.** Find the identity element for **'$*$'** on **A** if any**.**

(10) Test the commutativity , associativity of the binary operation **'$*$'** defined on $\mathbf{N \times N}$ as **:**
$\mathbf{(a \quad b) * (c \quad d) = (ac \quad bd)}$**.** Also find the identity element and inverse element if exist.

P.T.O

4 – Marks

(11) Show that the function $f : R_+ \to [-5 \ \infty)$ defined by $f(x) = 9x^2 + 6x - 5$ is invertible. Also find f^{-1} .

(12) Let $A = R - \{2\}$ and $B = R - \{3\}$. Consider the function $f : A \to B$ defined by $f(x) = \frac{3x+1}{x-2}$.
Show that ' f ' is invertible with $f^{-1}(x) = (2x + 1) / (x - 3)$

(13) If R_1 and R_2 are two equivalence relation , then show that $R_1 \cap R_2$ is also an equivalence relation.

(14) Show that the relation R in the set $A = \{ x : x \text{ is an integer } \& \ 0 \le x \le 12 \}$ as :
$R = \{ (a , b) : |a - b| = 4k \}$ with ' k' an integer, is an equivalence relation.

(15) Prove that the binary operation $*$ on N defined as $a * b = (a + b) / 2$, is commutative but not associative .

(16) If E and F are two independent events , prove that the events
(i) E and $F^{\prime}$ are also independent . (ii) $E^{\prime}$ and $F^{\prime}$ are also independent.

(17) Two cards are drawn at random without replacement from a well shuffled deck of 52 cards. Find the probability distribution of the number of aces.

(18) A and B throws a die alternately till one of them gets a '6' and wins the game. Find their respective probabilities of winning, if A starts the game.

(19) In a binomial distribution, the sum and product of the mean and variance are 25 / 3 and 50 / 3 respectively. Determine the binomial distribution.

(20) How many times must a man toss a fair coin so that the probability of having at least one head is more than 90% ?

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