ANIKET'S

OPERATION

MATHEMATICS

CBSE (XI) – 2017

(9 th Revised Edition)

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PREFACE

It is known to every student that **70 – 80 %** questions in **CBSE – XI** Exam have been asked from **NCERT** Text Book.

Though the remaining **20 – 30 % HOTS** (**H**igh **O**rder **T**hinking **S**kills) questions are creating furor in them.

As an outcome a lots of students had performed below the level , what they expect from themselves , in their **U.T's** or **Terminal Examinations**.

So, this '**Operation Mathematics CBSE XI – 2017'** has been designed for providing relief to such horrified students.

This package will not only bring confidence, but help the students in scoring the respectable **70 – 90 % Marks** in coming **CBSE XI – 2017 Exam.**

Aniket Manohar
Mob : 9013413543

1. Set Theory

(1) For three sets **A, B** and **C** prove that

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

(2) Draw venn – diagram of the following : **(i)** $A' \cup B'$ **(ii)** $(A \cup B)'$. **(iii)** $A - B$

(3) Using properties of sets, show that,

(i) $A \cap (A \cup B) = A$ **(ii)** $A \cup (A \cap B) = A$.

(iii) $(A - B) \cup B = A \Leftrightarrow B \subset A$. **(iv)** $A \cup B = A \cap B \Leftrightarrow A = B$.

(iv) $A - (B \cup C) = (A - B) \cap (A - C)$ **(v)** $(A \cup B) - C = (A - C) \cup (B - C)$

(4) For any sets **A** and **B,** show that $P (A \cap B) = P (A) \cap P (B)$.

(5) Let **A** and **B** be sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set **X,** show that $A = B$.

(6) Let **A, B** and **C** be the sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$. Show that $B = C$.

(7) There are **200** individuals with a skin disorder**, 120** had been exposed to the chemical C_1, **50** to chemical C_2, and **30** to both the chemicals C_1 and C_2.

Find the number of individuals exposed to **(i)** Chemical C_1 but not chemical C_2 **Ans : 90**

(ii) Chemical C_1 or chemical C_2. **Ans : 140**

(iii) No Chemical **Ans : 60**

(8) In a survey of **60** people**,** it was found that **25** people read newspaper **H, 26** read newspaper **T, 26** read newspaper **I, 9** read both **H** and **I, 11** read both **H** and **T, 8** read both **T** and **I, 3** read all three newspapers**.**

Find**: (i)** the number of people who read at least one of the newspapers**. Ans : 52**

(ii) the number of people who read exactly one newspaper**. Ans : 30**

(9) In a survey of **600** students in a school**, 150** students were found to be taking tea and **225** taking coffee**, 100** were taking both tea and coffee**.**

Find how many students were taking **(i)** neither tea nor coffee **? Ans : 325**

(ii) tea but not coffee**? Ans : 50**

(iii) at least one of the two drink**? Ans : 275**

(10) A college awarded **38** medals in football**, 15** in basketball and **20** in cricket. If these medals went to a total of **58** men and only three men got medals in all the three sports**,** how many received medals in exactly two of the three sports **? Ans : 9**

(11) In a survey it was found that **21** people liked product **A, 26** liked product **B** and **29** liked product **C.** If **14** people liked products **A** and **B, 12** people liked products **C** and **A, 14** people liked products **B** and **C** and **8** liked all the three products**.**

Find how many liked **(i)** exactly one product**. Ans : 20**

(ii) exactly two product**. Ans : 16**

(12) A survey shows that **63%** of Indians like coffee, whereas **76%** likes tea. If x **%** of Indians like both coffee and tea, find the range of possible values of x**. Ans :** $39 \le x \le 63$**.**

(13) In certain locality of a town of **10,000** families, it was found that **40%** families buy newspaper **A, 20 %** families buy newspaper **B** and **10%** families buy newspaper **C. 5%** families buy **A** and **B, 3%** families buy **B** and **C** and **4%** families buy **A** and **C.** If **2%** families buy all the three newspaper, find the number of families which buy**. (i) A** only **Ans : 3300**

(ii) B only **Ans : 1400**

(iii) None of **A, B** and **C. Ans : 4000**

(14) A class has **175** students. Following is the description showing the number of students studying one or more of the following subject in this class.

Mathematics **100,** Physics **70,** Chemistry **46,** Mathematics and Physics **30,** Mathematics and Chemistry **28,** Physics and Chemistry **23,** Mathematics, Physics and Chemistry **18.**

How many students are enrolled in **(i)** Mathematics alone **Ans : 60**

(ii) Physics alone **Ans : 35**

(iii) Chemistry alone **Ans : 13**

(iv) Are there students who have not offered any of these three subject**? Ans : 22**

(15) In a survey of **100** students, the number of students studying the various languages were found to be **:** English only **18,** English but not Hindi **23,** English and Sanskrit **8,** English **26,** Sanskrit **48,** Sanskrit and Hindi **8,** no language **24.**

Find **(i)** How many students were studying Hindi**?** **Ans : 18**

(ii) How many students were studying English and Hindi**?** **Ans : 3**

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2. Relations and Functions

(1) Find the domain and range of the following functions

(i) $\dfrac{1}{\sqrt{16-x^2}}$ *(ii)* $\dfrac{x^2+2x+1}{x^2-8x+12}$ *(iii)* $\sqrt{9-x^2}$

(iv) ${}^{7-x}P_{x-3}$ *(v)* $\dfrac{1}{1-x^2}$ *(vi)* $\sqrt{4x-x^2}$

Ans : (*i*) $(-4, 4)$; $[0.25, \infty)$ **(*ii*)** $R - \{2, 6\}$; $(-\infty, -3] \cup [0, \infty)$ **(*iii*)** $[-3 \quad 3]$; $[0 \quad 3]$

(*iv*) $\{3, 4, 5\}$; $\{1, 2, 3\}$ **(*v*)** $R - \{-1, 1\}$; $(-\infty, 0] \cup [1, \infty)$ **(*vi*)** $[0, 4]$; $[0 \quad 2]$

(2) **A = {1, 2, 3, 5}** and **B = {4, 6, 9}.** Let a relation **R : A → B ,** as $\{(x, y) : |x-y|$ is odd ; $x \in A, y \in B\}$.

(i) Write **R** in roster form. **(ii)** Find the domain of **R** **(iii)** Find the range of **R.**

Ans : (i) R = {(1 , 4) , (1 , 6) , (2 , 9) , (3 , 4) , (3 , 6) , (5 , 4) , (5 , 6) } **(ii)** {1, 2, 3, 5} **(iii)** {4, 6, 9}

(3) Let **R** be a relation from **N** to **N** defined by $R = \{(a, b) : a, b \in N$ and $a = b^2\}$. Are the following true?

(i) $(a, a) \in R$, for all $a \in N$ **(ii)** $(a, b) \in R \Rightarrow (b, a) \in R$ **(iii)** $(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R$.

Ans : (i) No **(ii)** No **(iii)** No

(4) Let **R** be a relation on **Z** defined by $R = \{(x, y) : |x-y|$ is divisible by n ; $x, y, n \in Z\}$.

Are the following true?

(i) $(x, x) \in R$, for all $x \in N$ **(ii)** $(x, y) \in R \Rightarrow (y, x) \in R$ **(iii)** $(x, y) \in R, (y, z) \in R \Rightarrow (x, z) \in R$.

Ans : (i) Yes **(ii)** Yes **(iii)** Yes

(5) Let **N** be the set of natural numbers and the relation **R** be defined on **N** such that $R = \{(x, y) : y = 2x, x, y \in N\}$. What is the domain, co-domain and range of **R?** Is this relation a function**?**

Ans : Domain **= N ;** Range = Even natural numbers **;** Co – domain **= N ;** Yes

(6) Let **A = {1, 2, 3, 4, 6}.** Let **R** be the relation on **A** defined by $\{(a, b) : a, b \in A, b$ is exactly divisible by $a\}$.

(i) Write **R** in roster form **(ii)** Find the domain of **R** **(iii)** Find the range of **R.**

Ans : (i) {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)} **(ii)** A **(iii)** A

(7) Let **A = {1, 2, 3,...,14}.** Define a relation **R** from **A** to **A** by $R = \{(x, y) : 3x - y = 0$, where $x, y \in A\}$.

Write down its domain, co-domain and range.

Ans : Domain = {1 , 2 , 3 , 4} **;** Co – domain = A **;** Range = {3 , 6 , 9 , 12}

(8) Let **N** be the set of natural numbers and the relation **R** be defined on **N** such that $R = \{(x, y) : x + 2y = 41 ; x, y \in N\}$. What is the domain, co-domain and range of **R?**

Is this relation a function**?**

Ans : Domain = {1 , 3 , 5 , – – – , 39} **;** Co – domain = N **;** Range = {1 , 2 , 3 , – – – , 20}**;** Yes

(9) Let **N** be the set of natural numbers and the relation **R** be defined on **N** such that $R = \{(a, b) : a + 3b = 12 ; a, b \in N\}$. What is the domain, co-domain and range of **R?**

Is this relation a function**?**

Ans : Domain = { 3 , 6 , 9} **;** Co – domain = N **;** Range = {1 , 2 , 3}**;** Yes

(10) Let **N** be the set of natural numbers and the relation **R** be defined on **N** such that $R = \{(a, b) : a + 2b = 10 ; a, b \in N\}$. Find

(i) domain of **R** and R^{-1} Ans : $D_R = \{2, 4, 6, 8\}$, $D_{R^{-1}} = \{1, 2, 3, 4\}$

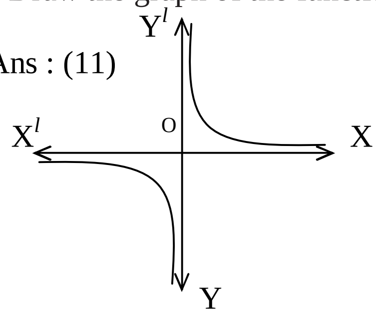
(ii) range of **R** and R^{-1} Ans : $R_R = \{1, 2, 3, 4\}$, $R_{R^{-1}} = \{2, 4, 6, 8\}$

(11) Draw the graph of the real valued function $f : \mathbf{R} - \{0\} \to \mathbf{R}$ defined by $f(x) = 1/x$.

(12) Draw the graph of the real valued function $f(x) = 1 - x \;;\; x < 0$
$= 1 \;;\; x = 0$
$= 1 + x \;;\; x > 0$

(13) Draw the graph of the function $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = x^3, x \in \mathbf{R}$.

Ans : (11)



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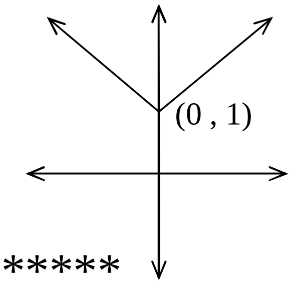

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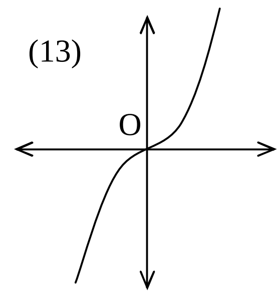

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3. Trigonometry

(1) Show that, $\cos^2 x + \cos^2\left\{x + \frac{\pi}{3}\right\} + \cos^2\left\{x - \frac{\pi}{3}\right\} = \frac{3}{2}$.

(2) Find the value of $\sin\left(\frac{x}{2}\right)$, $\cos\left(\frac{x}{2}\right)$ and $\tan\left(\frac{x}{2}\right)$ if $\sin x = -\frac{1}{4}$; x lies in IIIrd quadrant.

Ans: $\frac{\sqrt{6}}{3}, \frac{-\sqrt{3}}{3}, -\sqrt{2}$

(3) Find the value of $\tan\left(\frac{\pi}{8}\right)$ **Ans :** $\sqrt{2} - 1$

(4) Prove that; $2\cos\left(\frac{\pi}{13}\right) . \cos\left(\frac{9\pi}{13}\right) + \cos\left(\frac{3\pi}{13}\right) + \cos\left(\frac{5\pi}{13}\right) = 0$.

(5) Show that, $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

(6) Show that, $\tan 4x = \frac{4\tan x\,(1 - \tan^2 x)}{1 - 6\tan^2 x + \tan^4 x}$

(7) Find the general solution of the equation; $\sin x + \sin 3x + \sin 5x = 0$. **Ans :** $x = \frac{n\pi}{3}, n \in Z$

(8) Show that, $\cot x . \cot 2x - \cot 2x . \cot 3x - \cot 3x . \cot x = 1$

(9) Show that; $\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x . \cos 2x . \sin 4x$.

(10) Prove that : $\cos 20° . \cos 40° . \cos 60° . \cos 80° = 1/16$.

(11) Prove that : $\tan A + \tan\left\{A + \frac{\pi}{3}\right\} + \tan\left\{A + \frac{2\pi}{3}\right\} = 3.\tan 3A$

(12) Solve for 'x' : $\tan x + \sec x = \sqrt{3}$. **Ans :** $2n\pi + \frac{\pi}{6}; n \in Z$

(13) Show that : $\left\{1 + \cos\left(\frac{\pi}{8}\right)\right\}\left\{1 + \cos\left(\frac{3\pi}{8}\right)\right\}\left\{1 + \cos\left(\frac{5\pi}{8}\right)\right\}\left\{1 + \cos\left(\frac{7\pi}{8}\right)\right\} = \frac{1}{8}$.

(14) If α & β are roots of the equation : $a\cos\theta + b\sin\theta = c$, then prove that

(*i*) $\sin(\alpha + \beta) = \frac{2ab}{a^2 + b^2}$. (*ii*) $\tan(\alpha + \beta) = \frac{2ab}{a^2 - b^2}$ (*iii*) $\cos(\alpha + \beta) = \frac{a^2 - b^2}{a^2 + b^2}$.

(15) Prove that, $\sin^3 x + \sin^3\left\{x + \frac{2\pi}{3}\right\} + \sin^3\left\{x + \frac{4\pi}{3}\right\} = -\frac{3}{4}\sin 3x$

(16) Solve for x : $\tan\left\{x + \frac{\pi}{12}\right\} = 3\tan\left\{x - \frac{\pi}{12}\right\}$ **Ans :** $x = \frac{n\pi}{2} + (-1)^n\frac{\pi}{4}, n \in Z$

(17) Prove that : $3(\sin x - \cos x)^4 + 4(\sin^6 x + \cos^6 x) + 6(\sin x + \cos x)^2 = 13$

(18) If $\sin B = n\sin(B + 2A)$, then prove that : $(1 - n)\tan(A + B) = (1 + n) . \tan A$

(19) If $x + y = \pi/4$, prove that $(\cot x - 1)(\cot y - 1) = 2$.

(20) In **Δ ABC,** Prove that : $\cos^2\left(\frac{A}{2}\right) + \cos^2\left(\frac{B}{2}\right) + \cos^2\left(\frac{C}{2}\right) = 2 + 2\sin\left(\frac{A}{2}\right).\sin\left(\frac{B}{2}\right).\sin\left(\frac{C}{2}\right)$

(21) Prove that $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8} = \frac{3}{2}$

(22) Prove that $\tan 70^\circ = \tan 20^\circ + 2\tan 50^\circ$

(23) Prove that : $\cos\left(\frac{\pi}{5}\right).\cos\left(\frac{2\pi}{5}\right).\cos\left(\frac{4\pi}{5}\right).\cos\left(\frac{8\pi}{5}\right) = -\frac{1}{16}$

(24) Show that : $\frac{\sec 8x - 1}{\sec 4x - 1} = \frac{\tan 8x}{\tan 2x}$

(25) Prove that : $\cos 18^\circ = \frac{\sqrt{10 + 2\sqrt{5}}}{4}$

(26) Prove that : $\tan\frac{\pi}{24} = \sqrt{6} - \sqrt{3} - \sqrt{4} + \sqrt{2}$

(27) Prove that : $\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 16\theta}}}} = 2\cos\theta$

(28) If angle θ is divided into two parts such that the tangent of one part is n times the tangent of other, and δ is the difference of the two parts, then show that $\sin\theta = \frac{n+1}{n-1}\sin\delta$

(29) $\cos x \,.\, \cos\left(\frac{x}{2}\right) - \cos 3x \cos\left(\frac{9x}{2}\right) = \sin\left(\frac{7x}{2}\right).\sin 4x$

(30) If $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$, where α, β lie between **0** and $\frac{\pi}{4}$, then prove that $\tan 2\alpha = \frac{56}{33}$

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4. Principle of Mathematical Induction

(1) For all **n ≥ 1** , $1.3 + 3.5 + 5.7 + - - - + (2n - 1)(2n + 1) = \frac{n(4n^2 + 6n - 1)}{3}$

(2) For all **n ≥ 1** , **n (n + 1) (n + 5)** is a multiple of **3.**

(3) For all **n ≥ 1** , $(2n + 7) < (n + 3)^2$.

(4) For all **n ≥ 1** , $2.7^n + 3.5^n - 5$ is divisible by **24.**

(5) For all **n ≥ 1** , $1^2 + 2^2 + 3^2 + - - - + n^2 > n^3 / 3$

(6) For all **n ≥ 1** , $3^{2n+2} - 8n - 9$ is divisible by **8.**

(7) For all **n ≥ 1** , $2^{n-1} \leq n\,!$

(8) For all **n ≥ 1** , $n^3 + (n + 1)^3 + (n + 2)^3$ is divisible by **9.**

(9) For all **n ≥ 1** , $\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + - - - + \frac{1}{(2n + 1)(2n + 3)} = \frac{n}{6n + 9}$

(10) For all **n ≥ 1** , $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + - - - + \frac{1}{n(n + 1)(n + 2)} = \frac{n(n + 3)}{4(n + 1)(n + 2)}$

(11) For all **n ≥ 1** , $1 + \frac{1}{1 + 2} + \frac{1}{1 + 2 + 3} + - - - + \frac{1}{1 + 2 + - - - + n} = \frac{2n}{n + 1}$

(12) For all **n ≥ 1** , $\left(1 + \frac{3}{1}\right)\left(1 + \frac{5}{4}\right)\left(1 + \frac{7}{9}\right) - - \left(1 + \frac{2n + 1}{n^2}\right) = (n + 1)^2$.

(13) For all **n ≥ 1** , $1^2 + 3^2 + 5^2 + - - - + (2n - 1)^2 = \frac{n(2n - 1)(2n + 1)}{3}$

(14) For all **n ≥ 1** , $10^n + 3.4^{n+2} + 5$ is divisible by **9.**

(15) For all **n ≥ 1,** $1^3 + 3^3 + 5^3 + - - - + (2n - 1)^3 = n^2(2n^2 - 1)$

(16) For all $n \ge 1$, $\cos x + \cos 2x + \cos 3x + \cdots + \cos nx = \sin\left(\frac{nx}{2}\right).\text{cosec}\left(\frac{x}{2}\right).\cos\left(\frac{(n+1)x}{2}\right)$

(17) State and prove Binomial Theorem using Principle of Mathematical Induction.

(18) For all $n \ge 1$, $7 + 77 + 777 + 7777 + \cdots + n \text{ terms} = \frac{7}{9}\left\{\frac{10(10^n - 1)}{9} - n\right\}$

(19) For all $n \ge 1$, $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \cdots + n \text{ terms} = \frac{n(n+1)^2.(n+2)}{12}$

(20) For all $n \ge 1$, $1.3 + 2.3^2 + 3.3^3 + \cdots + n.3^n = \frac{(2n-1).3^{n+1} + 3}{4}$

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5. Complex Number

(1) Convert the following complex number in the polar form.

(i) $\frac{-16}{1 + i\sqrt{3}}$ **(ii)** $\frac{i - 1}{\cos(\pi/3) + i\sin(\pi/3)}$ **(iii)** $\frac{1 + 3i}{1 - 2i}$ **(iv)** $\frac{1 + 7i}{(2 - i)^2}$ **(v)** $\frac{1 + i}{1 - i} - \frac{1 - i}{1 + i}$

Ans : (i) $8(\cos 120^\circ + i\sin 120^\circ)$ **(ii)** $\sqrt{2}(\cos 75^\circ + i\sin 75^\circ)$ **(iii)** $\sqrt{2}(\cos 135^\circ + i\sin 135^\circ)$
(iv) $\sqrt{2}(\cos 135^\circ + i\sin 135^\circ)$ **(v)** $2(\cos 90^\circ + i\sin 90^\circ)$

(2) If $x - iy = \frac{\sqrt{a - ib}}{\sqrt{c - id}}$, prove that $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$

(3) Find the least integral value of **'m'** if $\left[\frac{1 + i}{1 - i}\right]^m = 1$. **Ans : m = 4**

(4) Find the real numbers **'x'** & **'y'** if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$. **Ans : x = 3 , y = – 3**

(5) Find real θ such that, $\frac{3 + 2i\sin\theta}{1 - 2i\sin\theta}$ is purely real. **Ans :** $\theta = n\pi, n \in Z$

(6) For any two complex number z_1 & z_2 prove that $Re(z_1 z_2) = Re\, z_1 . Re\, z_2 - Im\, z_1 . Im\, z_2$.

(7) If **'a'** and **'b'** are different complex numbers with $|b| = 1$, then find $\left|\frac{b - a}{1 - \bar{a}b}\right|$

(8) If $(x + iy)^3 = u + iv$, then show that $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$

(9) Solve for **'x'** : **(i)** $x^2 - (5 + i)x + (18 - i) = 0$. **Ans : 3 + 4i , 2 – 3i**
(ii) $x^2 - x + (1 + i) = 0$. **Ans :** i, $1 - i$
(ii) $2x^2 - (3 + 7i)x + (9i - 3) = 0$. **Ans :** $3i$, $(3 + i)/2$

(11) Find the square root of the complex number **(i)** $-7 - 24i$ **(ii)** $8 - 15i$ **(iii)** $8 + 6i$
Ans : (i) $\pm(3 + 4i)$ (ii) $\pm(5 - 3i)/\sqrt{2}$ (iii) $\pm(3 + i)$

(12) If $|z| = 1$, prove that $\frac{z - 1}{z + 1}$ is purely imaginary complex number.

(13) If $|v| = 1$, and $v = \frac{1 - zi}{z - i}$ then prove that z is purely real complex number.

(14) Prove that : $Re\{(1 - \cos x + 2i\sin x)^{-1}\} = 1/(5 + 3\cos x)$

(15) If $z = x + iy$, $z^{1/3} = a - ib$ and $bx - ay = k.ab(a^2 - b^2)$, then find the value of **'k'**. **Ans : 4**

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6. Inequations

(1) How many litres of water will have to be added to **1125** litres of the **45%** solution of acid so that the resulting mixture will contain more than **25%** but less than **30%** acid content **?** **Ans :** > 562.5 *l* & < 900 *l*

(2) A solution of **8%** boric acid is to be diluted by adding a **2%** boric acid solution to it . The resulting mixture is to be more than **4%** but less than **6%** boric acid**.** If we have **640** litres of the **8%** solution, how many litres of the **2%** solution will have to be added? **Ans :** > 320 *l* & < 1280 *l*

(3) A man wants to cut three lengths from a single piece of board of length **91cm .**The second length is to be **3cm** longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least **5cm** longer than the second ? **Ans :** ≥ 8cm and ≤ 22cm.

(4) A manufacturer has **600** litres of a **12%** solution of acid. How many litres of a **30%** acid solution must be added to it so that acid content in the resulting mixture will be more than **15%** but less than **18%?** **Ans :** > 120 *l* & < 300 *l*

(5) A plumber can be paid under two schemes given as**; Scheme – I : Rs 600** and **Rs 50** per hour**, Scheme – II : Rs 170** per hour**.** If the job takes **n hours,** for what values of **n** does the **scheme I** gives the plumber better wages ? **Ans :** < 5 hr

(6) The water acidity in a pool is considered normal when the average **pH** reading of three daily measurements is between **7.2** and **7.8.** If the first two **pH** readings are **7.48** and **7.85,** find the range of **pH** value for the third reading that will result in the acidity level being normal. **Ans :** > 6.27 **;** < 8.07

(7) Solve the inequation **:** $\left|\frac{3x-4}{2}\right| \le \frac{5}{12}$; $x \in R$ **Ans :** $[19/18 \quad 29/18]$

(8) Solve the inequation**:** $|2x-3| < |x+5|$; $x \in R$ **Ans :** $(-2/3 \quad 2)$

(9) Solve the inequation **:** $|x-2| + |x-3| \ge 6$; $x \in R$ **Ans :** $x \le -1/2$ **;** $x \ge 11/2$

<u>Solve the following inequations graphically</u>:

(10) $2x + y \le 12$, $4x + 5y > 20$, $x + 2y \le 12$, $x \ge 0$, $y \ge 0$

(11) $x + y \le 4$, $x + 5y \ge 4$, $6x + 2y \ge 8$, $x \le 3$, $y \le 3$, $x \ge 0$, $y \ge 0$

(12) $5x + 10y \le 50$, $x + y \ge 1$, $x - y < 0$, $y \le 4$, $x \ge 0$, $y \ge 0$

(13) $2x + 3y \ge 6$, $x - 2y \le 2$, $3x + 2y < 12$, $2y - 3x \le 3$, $x \ge 0$, $y \ge 0$

(14) $x + 2y \le 10$, $x + y \ge 1$, $x - y \le 0$, $x \ge 0$, $y \ge 0$

(15) $2x + y \ge 4$, $x + y \le 3$, $2x - 3y \le 6$, $x \ge 0$, $y \ge 0$

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7. Permutation & Combination

(1) Find the number of different signals that can be generated by arranging **at least 2** flags in order (one below the other) on a vertical staff, if five different flags are available. **Ans : 320**

(2) In how many ways can the letters of the word **PERMUTATIONS** be arranged if the
(i) vowels are all together, **Ans : 8! × 5! / 2!**
(ii) there are always **4** letters between **P** and **S ?** **Ans : 14 × 10 ! / 2!**

(3) Prove that : $^{n}C_{r} + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$.

(4) Find the number of words with or without meaning which can be made using all the letters of the word **AGAIN.** If these words are written as in a dictionary, what will be the **50th** word? **Ans : NAAIG**

(5) What is the number of ways of choosing **4** cards from a pack of **52** playing cards**?** In how many of these
(i) four cards are of the same suit, **Ans : 2860**
(ii) four cards belong to four different suits. **Ans : $(13)^4$.**

(6) From a class of **25** students**, 10** are to be chosen for an excursion party**.** There are **3** students who decide that either all of them will join or none of them will join**.** In how many ways can the excursion party be chosen**?**
Ans : $^{22}C_{10} + {}^{22}C_{7}$

(7) If the different permutations of all the letter of the word **EXAMINATION** are listed as in a dictionary,
(i) How many words are there in this list before the first word starting with **E?** **Ans : 10! / 4**
(ii) How many words are there in this list starting with a vowel**?** **Ans : 3×10! / 4**

(8) A committee of **7** has to be formed from **9** boys and **4** girls**.** In how many ways can this be done when the committee consists of **: (i)** at least **3** girls **?** **Ans : 588**
(ii) at most **3** girls **?** **Ans : 1632**

(9) Find the number of different 8-letter arrangements that can be made from the letters of the word **DAUGHTER** so that**, (i)** all vowels occur together**.** **Ans : 3! × 6!**
(ii) Respective position of vowel and consonant remains unchanged**.** **Ans : 3! × 5!**

(10) How many natural number not exceeding **4321** can be formed with the digits **1, 2, 3,** and **4,** if the digits can repeat**?** **Ans : 313**

(11) Determine the **5** card combination out of a deck of **52** cards if at least one of the **5** cards has to be king**?**
Ans : 886656

(12) In how many of the distinct permutations of the letters in **MISSISSIPPI** do the four **I's** not come together**?**
Ans : 33810

(13) If the letters of the word **SACHIN** are arranged according to the dictionary**,** find the order of **SACHIN.**
Ans : 601

(16) To fill **40** vacancies there are **100** candidates of which **15** are **SC, 10** are **ST** and **20** are **OBC.** If **15 %, 10 %** and **20 %** vacancies are reserved for **SC, ST** and **OBC** respectively**.** Find the number of ways in which the selection can be made**.** **Ans : $^{82}C_{22} \times {}^{15}C_{6} \times {}^{10}C_{4} \times {}^{20}C_{8}$**

(15) If a polygon has **27** diagonals, find the number of sides it can has**.** **Ans : 9**

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8. Binomial Theorem

(1) Show that $9^{n+1} - 8n - 9$ is divisible by **64,** whenever n is a positive integer.

(2) Using binomial theorem, prove that $6^n - 5n$ always leaves remainder **1** when divided by **25.**

(3) Find the middle term in the expansions of $\left(x/3 + 9y \right)^{10}$. **Ans :** $^{10}C_5 (3xy)^5$.

(4) The coefficients of the $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ terms in the expansion of $(x+1)^n$ are in the ratio **1 : 3 : 5.** Find **'n'** and **'r'.** **Ans : n = 7 , r = 3**

(5) The coefficients of three consecutive terms in the expansion of $(1+a)^n$ are in the ratio **1: 7 : 42 .** Find **n.** **Ans : n = 55**

(6) The second, third and fourth terms in the binomial expansion $(x+a)^n$ are **240, 720** and **1080,** respectively. Find **'x', 'a'** and **'n'.** **Ans : n = 5 , x = 2 , a = 3**

(7) If the coefficients of a^{r-1}, a^r and a^{r+1} in the expansion of $(1+a)^n$ are in **A.P,** prove that $n^2 - n(4r+1) + 4r^2 - 2 = 0$.

(8) Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(2^{1/4} + 3^{-1/4} \right)^n$ is $\sqrt{6} : 1$. **Ans : n = 10**

(9) Show that the middle term in the expansion of $(1+x)^{2n}$ is $\dfrac{1.3.5.7. \cdots (2n-1)\, 2^n . x^n}{n!}$; $n \in Z_+$.

(10) If **a** and **b** are distinct integers, using binomial theorem prove that **a – b** is a factor of $a^n - b^n$, whenever n is a positive integer.

(11) Find $(x+1)^6 + (x-1)^6$. Hence or otherwise evaluate $(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$.
Ans : $2(x^6 + 15x^4 + 15x^2 + 1)$ **; 198**

(12) If in the expansion of $(1+x)^n$, the coefficient of 5^{th} , 6^{th} and 7^{th} terms are in **A.P.** Find **n.** **Ans : n = 7 or 14**

(13) If the coefficients of 2^{nd}, 3^{rd} and 4^{th} in the expansion of $(1+x)^{2n}$ are in **A.P.** Prove that $2n^2 - 9n + 7 = 0$.

(14) Find the term independent of **x** in $\left(\dfrac{3x^2}{2} - \dfrac{1}{3x} \right)^9$ **Ans : x = 7 / 18**

(15) Find the 4^{th} term from the end in $\left(\dfrac{3}{x^2} - \dfrac{x^3}{6} \right)^7$ **Ans :** $35x^6 / 48$

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9. Sequence & Series

(1) The sum of **n** terms of two arithmetic progressions are in the ratio **(3n + 8):(7n + 15).** Find the ratio of their 12^{th} terms. **Ans : 7 : 16**

(2) The ratio of the sums of **m** and **n** terms of an **A.P.** is $m^2 : n^2$. Show that the ratio of m^{th} and n^{th} term is **(2m – 1) : (2n – 1).**

(3) If the sum of **n** terms of an **A.P.** is $3n^2 + 5n$ and its m^{th} term is **164,** find the value of **m.** **Ans : 27**

(4) The difference between any two consecutive interior angles of a polygon is 5°. If the smallest angle is 120°, find the number of the sides of the polygon. **Ans : 9**

(5) If $\dfrac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the **A.M** between the two numbers **a** & **b.** Then find the value of **n .**

(6) If $\dfrac{a^{n+1} + b^{n+1}}{a^n + b^n}$ is the **G.M** between the two numbers **'a'** & **'b'.** Then find the value of **n.**

(7) Sum of first **p, q, r** terms of an **A.P** are **a, b, c** respectively, then prove that ;
$\dfrac{a}{p}(q - r) + \dfrac{b}{q}(r - p) + \dfrac{c}{r}(p - q) = 0$.

(8) If the p^{th}, q^{th} and r^{th} terms of a **G.P.** are **a, b** and **c,** respectively. Prove that: $a^{q-r}.b^{r-p}.c^{p-q} = 1$.

(9) Find the sum of the sequence **7, 77, 777, 7777, - - -** to **n** terms. **Ans :** $\dfrac{7}{9}\left\{\dfrac{10(10^n - 1)}{9} - n\right\}$

(10) If the first and the n^{th} term of a **G.P.** are **'a'** and **'b',** respectively, and if **'P'** is the product of **n** terms, prove that ; $P^2 = (ab)^n$.

(11) If **a, b, c** and **d** are in **G.P.** show that : $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$.

(12) If **a, b, c, d** are in **G.P,** prove that $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in **G.P.**

(13) If **a, b, c** are in **G.P.** and $a^{1/x} = b^{1/y} = c^{1/z}$, prove that **x, y, z** are in **A.P.**

(13) If p^{th}, q^{th}, r^{th} and s^{th} terms of an **A.P** are in **G.P,** then show that **(p – q), (q – r), (r – s)** are also in **G.P.**

(14) Let **S** be the sum, **P** the product and **R** the sum of reciprocals of n terms in a **G.P.** Prove that $P^2 R^n = S^n$.

(15) If **a** and **b** are the roots of $x^2 - 3x + p = 0$ and **c, d** are roots of $x^2 - 12x + q = 0$, where **a, b, c, d** form a **G.P.** Prove that **(q + p) : (q – p) = 17:15**.

(16) The sum of three numbers in **G.P.** is **56.** If we subtract **1, 7, 21** from these numbers in that order, we obtain an arithmetic progression. Find the numbers. **Ans : 8, 16, 32**

(17) Find the sum of the first **n** terms of the series: **3 + 7 +13 +21 +31 + - - - .** **Ans :** $\dfrac{n^3 + 3n^2 + 5n}{3}$

(18) Find the sum to **n** terms of the series **: 5 + 11 + 19 + 29 + 41 - - - .** **Ans :** $\dfrac{n(n + 2)(n + 4)}{3}$

(19) Show that : $\dfrac{1 \times 2^2 + 2 \times 3^2 + - - - + n \times (n + 1)^2}{1^2 \times 2 + 2^2 \times 3 + - - - + n^2 \times (n + 1)} = \dfrac{3n + 5}{3n + 1}$.

(20) The sum of two numbers is **6** times their geometric means, show that numbers are in the ratio $\mathbf{3 + 2\sqrt{2} : 3 - 2\sqrt{2}}$

(21) Find the sum to **n** terms of the series, $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + - - -$ **Ans :** $\frac{n}{n+1}$

(22) If **a, b, c, d** and **p** are different real numbers such that, $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$, then show that **a, b, c** and **d** are in **G.P.**

(23) If **p, q, r** are in **G.P.** and the equations, $px^2 + 2qx + r = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then show that **d / p , e / q , f / r** are in **A.P**

(24) If $a\left(\frac{1}{b} + \frac{1}{c}\right)$; $b\left(\frac{1}{c} + \frac{1}{a}\right)$; $c\left(\frac{1}{a} + \frac{1}{b}\right)$ are in **A.P.** Prove that **a , b , c** are in **A.P.**

(25) The ratio of the **A.M.** and **G.M.** of two positive numbers **a** and **b,** is **m : n.** Show that $a : b = m + \sqrt{m^2 - n^2} : m - \sqrt{m^2 - n^2}$

(26) If **a, b, c** are in **A.P.; b, c, d** are in **G.P.** and $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in **A.P.** prove that **a, c, e** are in **G.P.**

(27) Find the sum of the series up to **n** terms : $1^3 + \frac{1^3 + 2^3}{1 + 3} + \frac{1^3 + 2^3 + 3^3}{1 + 3 + 5} + - - - .$ **Ans :** $\frac{2n^3 + 9n^2 + 13n}{24}$

(28) **150** workers were engaged to finish a job in a certain number of days**. 4** workers dropped out on second day**, 4** more workers dropped out on third day and so on**.** It took **8** more days to finish the work**.** Find the number of days in which the work was completed**.** **Ans : 25**

(29) If the p^{th}, q^{th} and r^{th} terms of an **A.P.** as well as a **G. P.** are **a, b** and **c,** respectively**.**
Prove that**:** $a^{b-c} . b^{c-a} . c^{a-b} = 1.$

(30) If **a** is the **A.M** of **b** and **c** and the two geometric means are G_1, and G_2, then prove that $G_1^3 + G_2^3 = 2abc$.

(31) If ***p*** and ***q*** are the two **A.Ms** between two numbers ***a*** and ***b*** and the geometric means between them is ***G.*** Then prove that $G^2 = (2p - q)(2q - p)$.

(32) If **a, b, c** are in **AP** prove that $\frac{1}{\sqrt{b} + \sqrt{c}}$, $\frac{1}{\sqrt{c} + \sqrt{a}}$, $\frac{1}{\sqrt{a} + \sqrt{b}}$ are also in **A.P**

(33) 300 trees are planted in a regular pattern in rows in the shape of isosceles triangle, the number in the successive rows diminishing by one from the base to the apex. How many trees are there in the row which forms the base of the triangle? ***Ans :*** 24

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10. Straight Lines

(1) If **p** and **q** are the lengths of perpendiculars from the origin to the lines $x\cos\theta - y\sin\theta = k\cos 2\theta$ and $x\sec\theta + y\,\mathrm{cosec}\,\theta = k$, respectively, prove that $p^2 + 4q^2 = k^2$.

(2) Find the equation of the line passing through the point **(2 , 2)** and cutting off intercepts on axes whose sum is **9.** **Ans : 3x + 6y = 18 ; 6x + 3y = 18**

(3) Find the foot of the perpendicular drawn from the point **(–1 , 3)** to the line $3x - 4y - 16 = 0$.
Ans : (68 / 25 , – 49 / 25)

(4) Assuming that straight lines work as the plane mirror for a point, find the image of the point **(1, 2)** in the line $x - 3y + 4 = 0$. **Ans : (6/ 5 , 7 / 5)**

(5) Prove that area of the triangle formed by the lines $y = m_1 x + c_1$; $y = m_2 x + c_2$; $x = 0$ is $\dfrac{(c_1 - c_2)^2}{2|m_1 - m_2|}$

(6) A line is such that its segment between the lines **5x – y + 4 = 0** and **3x + 4y – 4 = 0** is bisected at the point **(1, 5).** Obtain its equation. **Ans : 107x – 3y = 92**

(7) Find the value of **p** so that the three lines **3x + y – 2 = 0, px + 2 y – 3 = 0** and **2x – y – 3 = 0** may be concurrent (intersecting at one point) . **Ans : 5**

(8) If three lines $y = m_1 . x + c_1$; $y = m_2 . x + c_2$ and $y = m_3 . x + c_3$ are concurrent, prove that $m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$.

(9) Show that distance of the point **(1 , 2)** from the line **4x + 7y + 5 = 0** along the line **2x – y = 3** is $23\sqrt{5} / 18$

(10) If the line joining two points **A(2, 0)** and **B(3, 1)** is rotated about A in anticlockwise direction through an angle of $\frac{\pi}{12}$. Find the equation of the line in new position. Ans : $\sqrt{3}\,x - y - 2\sqrt{3} = 0$

(11) Find perpendicular distance from the origin of the line joining the points **(cosθ, sin θ)** and **(cos ϕ, sin ϕ).**
Ans : $\cos\left\{\dfrac{\theta - \phi}{2}\right\}$

(12) Find the distance of the line **4x – y = 0** from the point **P (4, 1)** measured along the line making an angle of **135°** with the positive x-axis. **Ans :** $3\sqrt{2}$

(13) Find the direction in which a line must be drawn through the point **(–1 , 2)** so that its point of intersection with the line **x + y = 4** may be at a distance **3 units** from this point . **Ans** : Slope = **0**

(14) Find the equation of the line through the point **(3 , 2)** and which makes an angle 45° with **x – 2y = 3.**
Ans : 3x – y = 7 ; x + 3y = 9

(15) A ray of light passing through the point **(1 , 2)** reflects on the x – axis a the point A and the reflected ray passes through point **(5 , 3),** then find the coordinate of the point A. **Ans : (13 / 5 , 0)**

(16) Two consecutive sides of a parallelogram are **4x + 5y = 0** and **7x + 2y = 0.** If the equation of one diagonal is **11x + 7y = 9,** find the equation of the other diagonal. **Ans : y = x**

(17) If the line $\frac{x}{a} + \frac{y}{b} = 1$ moves in such a way that $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$ where *c* is a constant. Then prove that the locus of the foot of the perpendicular from the origin on the given line is $x^2 + y^2 = c^2$

(18) If the slope of a line passing through the point **A(3, 2)** is $\frac{3}{4}$, then find points on the line which are **5** units away from the point A. **Ans : (–1, –1), (7, 5)**

(19) At white point the origin must be shifted so that the coefficients of *x* and *y* in the new equation obtained from $x^2 + y^2 + 2x + 4y - 2 = 0$ is **0.** **Ans : (–1 , –2)**

(20) Two vertices of a triangle are **(3 , –1)** and **(–2 , 3)** and its orthocenter is at origin**.** Find the coordinates of the third vertex**.** **Ans : (–36 / 7 , –45 / 7)**

(21) Find the equation of the straight line passing through the point **(–2 , –7)** and having an intercept of length **3** between the straight lines **4x + 3y = 12** and **4x + 3y = 3.** **Ans : x + 2 = 0 , 7x + 24y + 182 = 0**

(22) A ray of light is sent along the line **x – 2y = 3.** Upon reaching the line **3x – 2y = 5,** the ray is reflected from it. Find the equation of the line containing the reflected ray**.** **Ans : 29x – 2y = 31.**

(23) Find the locus of the centers of circles touching the straight lines **3x – 4y + 7 = 0** and **12x – 16y + 52 = 0.** **Ans : 3x – 4y + 10 = 0**

(24) A variable line which always remains at a constant distance **' *p*'** from origin, cuts the coordinate axes at **A, B** respectively**.** Prove that the locus of the point of intersection of the lines drawn parallel to the coordinate axes through **A** and **B** is $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{p^2}$.

(25) A variable line which always remains at a constant distance **'3*p*'** from origin, cuts the coordinate axes at **A, B** respectively**.** Prove that the locus of the centroid of the triangle **OAB** is $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{p^2}$.

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11. Conic – Section

(1) Find the equation of the circle which passes through the point **(4 , 1)** and **(6 , 5)** and whose centre lies on the line **4x + y = 16 .** **Ans : $x^2 + y^2 – 6x – 8y + 15 = 0$**

(2) Find the equation of the circle with radius **5units,** whose centre lies on the **X – axis** and which passes through the point **(2 , 3).** **Ans : $x^2 + y^2 + 4x – 21 = 0$ & $x^2 + y^2 – 12x + 11 = 0$**

(3) Find the equation of the circle passing through origin and making intercepts **'a'** and **'b'** on the coordinate axes**.** **Ans : $x^2 + y^2 – ax – by = 0$**

(4) Find the equation of the circle whose centre is **(3 , –1)** and which cut off an intercept of length **6** from the line **2x – 5y + 18 = 0.** **Ans : $x^2 + y^2 – 6x + 2y – 28 = 0$**

(5) Find the intercept on axes made by a circle having **(–4 , 3)** and **(12 , –1)** as ends of a diameter. **Ans : 2√67, 4√13**

(6) Find **focus, axis,** the equation of the **directrix,** and length of the **latus rectum** of the parabola **$x^2 = –9y$.** **Ans : (0 , –9/4) ; y – axis , 4y = 9 , Latus rectum = 9**

(7) Find the **vertex, focus, LLR, axis** and **directrix** of the parabola : **$4y^2 + 12x – 20y + 67 = 0$** **Ans : (–7 / 2 , 5 / 2) ; (–17 / 4 , 5 / 2) ; 3 , 2y = 5, 4x + 11 = 0**

(8) Find the **foci, vertices, eccentricity,** and the length of the **latus rectum** of the ellipse **$16x^2 + y^2 = 16$.** **Ans : (0 , ± √15 / 4) ; (0 , ± 4) , e = √15 / 4 , 1 / 2**

(9) Find the centre**,** the lengths of axes**,** eccentricity**,** foci of the ellipse**: $25x^2 + 9y^2 – 150x – 90y + 225 = 0$.** **Ans : (3 , 5) ; 10 , 6 ; e = 4 / 5 ; (3 , 1) , (3 , 9)**

(10) Find the coordinates of **foci and vertices, the eccentricity** and the length of **latus rectum** of the hyperbola **$9y^2 – 4x^2 = 36$.** **Ans : (0 , ± √13) ; (0 , ± 2) ; e = √13 / 2 ; 9**

(11) Find the centre**,** the lengths of axes**,** eccentricity**,** foci of the hyperbola**: $x^2 – 2y^2 – 2x + 8y – 1 = 0$.** **Ans : (1 , 2); T.A: 2√3 , C.A:2√6; e = √3 ; (1 , 5) , (1 , –1)**

(12) Find the equation of hyperbola having foci **(± 4 , 0)** and the length of **latus rectum** is **12 .** **Ans : $3x^2 – y^2 = 12$**

(13) Find the equation of the hyperbola having foci on **$(0, \pm\sqrt{10})$** and which passes through **(2 , 3).**

Ans : $x^2 - y^2 = 5$

(14) Find the equation of conic – section such that**, $e = 3/4$,** foci on **y – axis,** centre at origin and passing through the point **(6 , 4).** **Ans : $16x^2 + 7y^2 = 688$**

(15) Find the equation of the ellipse, such that major axis is **x – axis,** centre is at origin and the ellipse passes through **(4 , 3) and (6 , 2).** **Ans : $x^2 + 4y^2 = 52$**

(16) The cable of uniform loaded suspension bridge hangs in the form of a parabola**.** The roadway is horizontal and **100m** long is supported by vertical wire attached to the cable**,** the longest wire being **30m** and the shortest wire being **6m .** Find the length of the wire attached to the roadway **18m** from the middle**.** **Ans : 9.11 m (approx.)**

(17) An arch is in the form of a semi – ellipse is **8m** wide and **2m** high at the centre**.** Find the height of arch at a point **1.5** from one end**.** **Ans : 1.56m (approx.)**

(18) A man is running on a racecourse notes that the sum of the distances from the two flag posts from him is always **10m** and the distance between the flag posts **8m.** Find the equation of the path traced out by the man**.** **Ans : $9x^2 + 25y^2 = 225$**

(19) A rod **AB = 15 cm** lies in between coordinate axes in such a way **A** always lies on **x – axis** and **B** on **y – axis .** Prove that locus of a point on the rod which divides **AB** in the ratio **3 : 2.** is **$81x^2 + 36y^2 = 2916$**

(20) A equilateral triangle is inscribed in the parabola **$y^2 = 4ax$,** where one vertex is at the vertex of the parabola**.** Find the length of sides of the equilateral triangle **.** **Ans : $8\sqrt{3} \times a$**

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12. Three – Dimension

(1) Find the coordinates of the point which divides the line segment joining the points **(1, –2, 3)** and **(3, 4, –5)** in the ratio **2 : 3 (i)** internally, and **(ii)** externally. **Ans : (i) (9 / 5, 2 / 5 , –1 / 5) (ii) (–3, –14, 19)**

(2) Three vertices of a parallelogram **ABCD** are **A(3, – 1, 2), B (1, 2, – 4)** and **C(– 1, 1, 2).** Find the coordinates of the fourth vertex**.** **Ans : (1, –2, 8)**

(3) Find the equation of the set of points which are equidistant from the points **(1, 2, 3)** and **(3, 2, –1).**

Ans : x = 2z

(4) Find the equation of the set of points P, the sum of whose distances from **A (4, 0, 0)** and **B(– 4, 0, 0)** is equal to **10.** **Ans : $9x^2 + 25y^2 + 25z^2 = 225$**

(5) Using section formula, prove that the three points **A(– 4, 6, 10), B(2, 4, 6)** and **C (14, 0, –2)** are collinear. Also find the ratio in which **C** divides **AB .** **Ans : externally 3 : 2**

(6) Find the coordinates of the centroid of the triangle whose vertices are **(x_1, y_1, z_1), (x_2, y_2, z_2)** and **(x_3, y_3, z_3).**

(7) Find the ratio in which the line segment joining the points **(4, 8, 10)** and **(6, 10, – 8)** is divided by the YZ – plane **(x = 0).** **Ans : externally 2 : 3**

(8) Find the coordinates of the point where the line through **(3, – 4, – 5)** and **(2, – 3, 1)** crosses the plane **2x + y + z = 7 .** **Ans : (1 , – 2 , 7)**

(9) Show that the points **A (1, 2, 3), B (–1, –2, –1), C (2, 3, 2)** and **D (4, 7, 6)** are the vertices of a parallelogram **ABCD,** but it is not a rectangle**.**

(10) Verify that **(0, 7, 10), (–1, 6, 6)** and **(– 4, 9, 6)** are the vertices of a right angled triangle.

(11) Find the lengths of the medians of the triangle with vertices **A (0, 0, 6), B (0, 4, 0)** and **C(6, 0, 0).**

Ans : 7 , $\sqrt{34}$, 7

(12) Find the coordinates of a point on **y-axis** which are at a distance of **$5\sqrt{2}$** from the point **P (3, –2, 5).**

Ans : (0, 2, 0) ; (0, –6, 0)

(13) A point **R** with x-coordinate **4** lies on the line segment joining the points **P(2, –3, 4)** and **Q (8, 0, 10).** Find the coordinates of the point **R.** **Ans : (4, –2, 6)**

(14) Show that the points **(–2, 3, 5), (1, 2, 3)** and **(7, 0, –1)** do not forms a triangle.

(15) If **A(3, 2, 0), B(5, 3, 2), C(–9, 6, –3)** are the vertices of a triangle Find the length **AD,** if **AD** bisects the angle $\angle$ **BAC.** **Ans :** $\frac{13\sqrt{6}}{16}$

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13. Limits & Derivatives

(1) Find $\lim_{x \to 0} f(x)$, where the function is $f(x) = \frac{|x|}{x}$; $x \neq 0$

$= 0$; $x = 0$ **Ans :** doesn't exist

(2) Find $\lim_{x \to 0} f(x)$, where the function is $f(x) = 2x + 3$; $x \leq 0$

$= 3(x + 1)$; $x > 0$ **Ans : 3**

(3) If $\lim_{x \to 1} f(x) = f(1)$, where the function $f(x) = \begin{cases} a + bx & ; x < 1 \\ 4 & ; x = 1 \\ b - ax & ; x > 1 \end{cases}$, Find the value of **a** and **b.**

Ans : a = 0 , b = 4

(4) If $\lim_{x \to a} f(x)$ exists, where the function is $f(x) = \begin{cases} |x| + 1 & ; x < 0 \\ 0 & ; x = 0 \\ |x| - 1 & ; x > 0 \end{cases}$, Find the value of **a**

Ans : a $\neq$ 0

(5) If $\lim_{x \to 0} f(x)$ and $\lim_{x \to 1} f(x)$ both exists, where the function, $f(x) = \begin{cases} mx^2 + n & ; x < 0 \\ m + nx & ; 0 \leq x \leq 1 \\ m + nx^3 & ; 1 < x \end{cases}$

Find the possible integral values of **m** and **n.** **Ans : m = n ; $\forall$ m , n $\in$ Z**

(6) Prove that : $\lim_{x \to 0} (\operatorname{cosec} x - \cot x) = 0$

(7) Prove that : $\lim_{x \to 1} \left\{ \frac{x - 2}{x^2 - x} - \frac{1}{x^3 - 3x^2 + 2x} \right\} = 2$

(8) Prove that : $\lim_{x \to \pi/2} \left\{ \frac{2\tan 2x}{2x - \pi} \right\} = 2$

(9) Prove that : $\lim_{x \to 3} \left\{ \frac{x^4 - 81}{2x^2 - 5x - 3} \right\} = \frac{108}{7}$

(10) Prove that : $\lim_{x \to 1} \left\{ \frac{x^4 - 3x^3 + 2}{x^3 - 5x^2 + 3x + 1} \right\} = \frac{5}{4}$

(11) Prove that : $\lim_{x \to a} \left\{ \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}} \right\} = 2\sqrt{a} \cdot \cos a$

(12) Prove that : $\lim_{x \to \pi/4} \left\{ \frac{4(\sin x - \cos x)}{4x - \pi} \right\} = \sqrt{2}$

(13) Prove that : $\lim_{x \to 1} \left\{ \frac{\sqrt{1 + 2x} - x\sqrt{3}}{\sqrt{3 + x} - 2\sqrt{x}} \right\} = \frac{8}{3\sqrt{3}}$

(13) Find $\lim_{x \to 0} f(x)$ of the function $f(x) = x . \sin(1/x)$; $x \neq 0$
$= 0$; $x = 0$. **Ans : 0**

(14) Find the values of **'k'** so that for the function $f(x) = \frac{k . \cos x}{\pi - 2x}$; $x \neq \pi/2$
$= 3$; $x = \pi/2$,

$\lim_{x \to \pi/2} f(x) = f(\pi/2)$. **Ans : k = 6**

Find the derivative of the following functions from first principle:

(15) $\sin x + \cos x$ (16) $\frac{2x + 3}{x - 2}$ (17) $x \sin x$ (18) $(\cot x)^{1/3}$

(19) $\sqrt{\sin x}$ (20) $\tan \sqrt{x}$ (21) $\operatorname{cosec}(5x + 3)$

Find the derivative of the following functions:

(22) $\frac{x^5 - \cos x}{\sin x}$ (23) $\frac{x}{\sin^n x}$ (24) $\frac{\sin x + \cos x}{\sin x - \cos x}$ (25) $\frac{x \cos x}{x - \tan x}$

Answer

(15) $\cos x - \sin x$ **(16)** $-7/(x-2)^2$ **(17)** $x \cos x + \sin x$

(18) $-(\operatorname{cosec}^2 x . \cot^{-2/3} x)/3$ **(19)** $\cos x / 2\sqrt{\sin x}$ **(20)** $(\sec^2 \sqrt{x}) / 2\sqrt{x}$

(21) $-5\operatorname{cosec}(5x + 3).\cot(5x + 3)$ **(22)** $\frac{1 + 5x^4 \sin x - x^5 \cos x}{\sin^2 x}$ **(23)** $\frac{1 - nx \cot x}{\sin^n x}$

(24) $\frac{-2}{(\sin x - \cos x)^2}$ **(25)** $\frac{\sin x (x \tan x - x^2 - 1) + x \sec x}{(x - \tan x)^2}$

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14. Statistics

(1) Find mean deviation about the mean for the following data :

x_i	2	5	6	8	10	12
f_i	2	8	10	7	8	5

Ans : 2 . 3

(2) Find the mean deviation about the median for the following data:

xi	3	6	9	12	13	15	21	22
fi	3	4	5	2	4	5	4	3

Ans : 4.97

(3) Find the mean deviation about the mean for the following data :

Marks obtained	10-20	20-30	30 - 40	40-50	50-60	60-70	70-80
Number of students	2	3	8	14	8	3	2

Ans : 10

(4) Calculate the mean deviation about median age for the age distribution of **100** persons given below**:**

Age	16 – 20	21 – 25	26 – 30	31 – 35	36 – 40	41 – 45	46 – 50	51 – 55
Number	5	6	12	14	26	12	16	9

Ans : 7.35

(5) Calculate mean, Variance and Standard Deviation for the following distribution.

Classes	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
Frequency	3	7	12	15	8	3	2

Ans : 62, 201, 14.18

(6) Calculate Standard Deviation, using short-cut method for the following distribution**:**

Classes	0 – 30	30 – 60	60 – 90	90 – 120	120 –150	150 –180	180 -210
Frequency	2	3	5	10	3	5	2

Ans : 107, 2276 , 47.707

(7) Find the mean, variance and standard deviation using short-cut method.

x_i	60	61	62	63	64	65	66	67	68
f_i	2	1	12	29	25	12	10	4	5

Ans : 64, 2.85 , 1.69

(8) Find the mean and variance standard deviation for the frequency distributions

Classes	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	5	8	15	16	6

Ans : 27, 132, 11.489

(9) From the data given below state which group is more variable**, A** or **B?**

Marks	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Group A	9	17	32	33	40	10	9
Group B	10	20	30	25	43	15	7

Ans : B

(10) The following is the record of goals scored by team A in a football session:

No. of goals	0	1	2	3	4
No. of matches	1	9	7	5	3

For the team **B,** mean number of goals scored per match was **2** with standard deviation **1.25** goals**.** Find which team may be considered more consistent?

Ans : A

(11) The mean and standard deviation of **20** observations are found to be **10** and **2,** respectively**.** On rechecking**,** it was found that an observation **8** was incorrect**.** Calculate the correct mean and standard deviation in each of the following cases **: (i)** If wrong item is omitted**.** **Ans : 10.1, 1.99**

(ii) If it is replaced by **12.** **Ans : 10.2, 1.98**

(12) The mean and variance of eight observations are **9** and **9.25,** respectively**.** If six of the observations are **6, 7, 10, 12, 12** and **13,** find the remaining two observations**.** **Ans : 4 , 8**

(13) The mean and standard deviation of **100** observations were calculated as **40** and **5.1,** respectively by a student who took by mistake **50** instead of **40** for one observation. What are the correct mean and standard deviation**?** **Ans : 39.9 , 5**

(14) The mean and standard deviation of a group of **100** observations were found to be **20** and **3,** respectively**.** Later on it was found that three observations were incorrect, which were recorded as **21, 21** and **18.** Find the mean and standard deviation if the incorrect observations are omitted**.** **Ans : 20, 3.036**

(15) The mean and variance of **7** observations are **8** and **16,** respectively**.** If five of the observations are **2, 4, 10, 12, 14.** Find the remaining two observations. **Ans : 6, 8**

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15. Probability

(1) If **'A'** , **'B'** and **'C'** are any three events associated with any random experiment, then prove that, $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$.

(2) A fair coin is tossed **4** times a person win **Re1,** for each head and lose **Rs1.50** for each tail that turns up**.** From the sample space & calculate how many different amount of money the person can have after **4** tosses also calculate the probability of having each of these amount**.**

Ans : P(Gain Rs 4) = 1 / 16 ; P(Gain Rs 1.50) = 1 / 4 ; P(Loss Re 1) = 3 / 8 ; P(Loss Rs 3.50) = 1 / 4 , P(Loss Rs 6) = 1 / 16

(3) If **'A'** and **'B'** are any two events such that $P(A) = 0.42$, $P(B) = 0.48$ & $P(A \cap B) = 0.16$. Determine

(i) P(not **A)** **(ii) P(** not **B)** **(iii) P(A** or **B)** **(iv) P(** not **A** and not **B).**

Ans : (i) 0.58 (ii) 0.52 (iii) 0.74 (iv) 0.26

(4) Find the probability that when a hand of **7** cards is drawn from a well shuffled deck of **52** cards it contains

(i) all kings **(ii)** exactly **3** kings. **(iii)** at least **3** kings.

Ans : (i) 1 / 7735 (ii) 9 / 1547 (iii) 46 / 7735

(5) Out of **100** students**,** two sections of **40** and **60** are formed**.** If you and your friend are among 100 students**.** What is the probability that**, (i)** you both enter the same section **?** **Ans : 17 / 33**

(ii) you both enter the different section **?** **Ans : 16 / 33**

(6) In Class **XI** of a school **40%** of the students study Mathematics and **30%** study Biology**. 10%** of the class study both Mathematics and Biology**.** If a student is selected at random from the class**,** find the probability that he will be studying Mathematics or Biology**.** **Ans : 0.6**

(7) A card is drawn at random from a pack of 52 playing cards. Find the probability of getting a king or a heart or a red card**.** ***Ans* : 7 / 13.**

(7) In a class of **60** students**, 30** opted for **NCC, 32** opted for **NSS** and **24** opted for both **NCC** and **NSS.** If one of these students is selected at random**,** find the probability that

(i) The student has opted neither **NCC** nor **NSS.** **Ans : 11 / 30**

(ii) The student has opted **NSS** but not **NCC.** **Ans : 2 / 15**

(8) On her vacations Veena visits four cities **(A, B, C** and **D)** in a random order**.** What is the probability that she visits **(i) A** before **B ?** **Ans : 1 / 2**

(ii) A just before **B ?** **Ans : 1 / 4**

(iii) A before **B** and **B** before **C?** **Ans : 1 / 6**

(9) Three letters are dictated to three persons and an evelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope. **Ans :**

(10) If **4**-digit numbers greater than or equals to **5,000** are randomly formed from the digits **0, 1, 3, 5,** and **7,** what is the probability of forming a number divisible by **5** when,

(i) The digits are repeated **?** **Ans : 2 / 5**

(ii) the repetition of digits is not allowed **?** **Ans : 3 / 8**

(11) Four letters are dictated to four persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter.
Find the probability that. **(i)** Exactly one letter is in its proper envelope. **Ans : 1 / 3**

(ii) Exactly two letters is in its proper envelope. **Ans : 1 / 4**

(iii) No letter is in its proper envelope. **Ans : 9 / 24**

(iv) All letters are not in its proper envelope. **Ans : 23 / 24**

(12) Two students Shivani and Swati appeared in an examination. The probability that Shivani will qualify the examination is **0.05** and that Swati will qualify the examination is **0.10.** The probability that both will qualify the examination is **0.02.** Find the probability that,

(i) Both Shivani and Swati will not qualify the examination. **Ans : 0.98**

(ii) At least one of them will not qualify the examination. **Ans : 0.87**

(iii) Only one of them will qualify the examination. **Ans : 0.11**

(13) In a bag **A** there are **5** white, **8** red balls; in bag **B** there are **7** white, **6** red balls and in bag **C** there are **6** white and **5** red balls . One ball is taken out at random from each bag. Find the probability that all three balls are of the same colour. **Ans : 450 / 1859**

(14) In a bag **A** there are **5** white, **4** red balls; in bag **B** there are **7** white, **2** red balls and in bag **C** there are **4** white and **5** red balls. One bag is selected and two balls are drawn at random from the bag. Find the probability that the balls drawn are red . **Ans : 17 / 108**

(15) In a bag **A** there are **5** white, **4** red balls; in bag **B** there are **7** white, **2** red balls. One ball is transferred from the bag **A** to the bag **B** and then a ball is drawn from the bag **B.** Find the probability that the ball drawn from bag **B** is white. **Ans : 34 / 45**

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